UNIVERSITI TEKNOLOGI MALAYSIA
PEPERIKSAAN SEMESTER I
SESI 2000/2001

KOD MATAPELAJARAN	: **SEE 4113**
MATAPELAJARAN	: **SISTEM KAWALAN MODEN**
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TARIKH	: **22 OKTOBER 2000**

ARAHAN KEPADA CALON : **JAWAB EMPAT (4) SOALAN SAHAJA**

KERTAS SOALAN INI TERDIRI DARIPADA **5** MUKA SURAT SAHAJA

Q1. Fiqure Q1 shows a mechanical system consisting of mass M_1 and M_2, damper constant B and spring stiffness K_1 and K_2. When force $f(t)$ acts on mass M_1, it moves to position $x_1(t)$ while mass M_2 moves to position $x_2(t)$. Let $x_2(t)$ be the output. Find state-space representation of the system using $x_1(t)$, $x_2(t)$ and their first derivatives as state-variables.

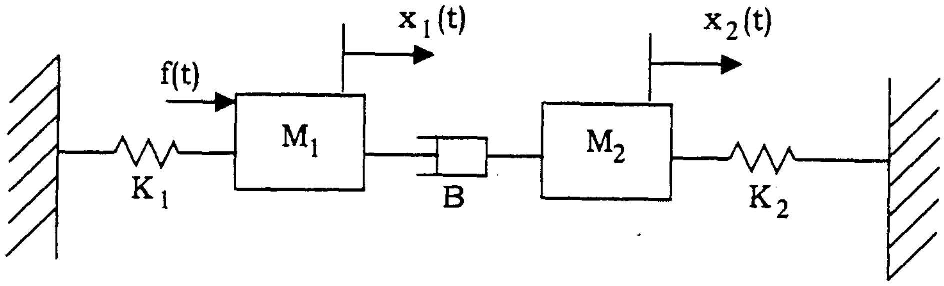


Figure Q1

Q2. A control system is defined by the following state-space equations:

$$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} -5 & -1 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 2 \\ 5 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

a) Is the system stable? Give reasons for your answer.

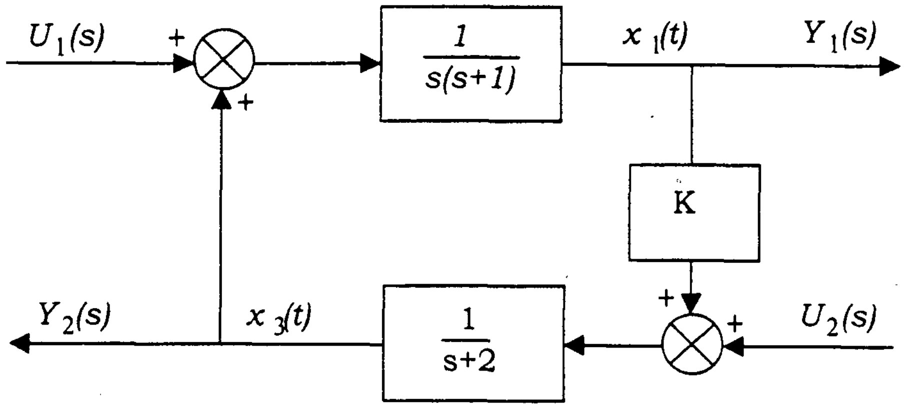
b) Determine the state transition matrix, $\Phi(t)$.

c) Solve the state equation for $x(t)$ if the system is subjected to an input $u(t)=e^{-t}$. Assume zero initial state vector.

Q3. A two-input two-output system has the structure as shown in Figure Q3 where $U_1(s)$ and $U_2(t)$ are the inputs while $Y_1(s)$ and $Y_2(s)$ are the outputs.

a) Using the states $x_1(t)$, $x_2(t)=\dot{x}_1(t)$ and $x_3(t)$ as defined in the diagram, determine the state and output equations for the system in vector-matrix form.

b) Determine the transfer function $\dfrac{Y_1(s)}{U_2(s)}$



Q4. A system is represented in the following state-space representation.

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 20.6 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

a) Determine the gain of the state-variable feedback controller so that the poles of the closed-loop system are located at (-1.8+j2.4) and (-1.8-j2.4).

b) Determine the gain of the observer matrix which has both eigenvalues located at –8.

Q5 A system is described by the following state-space equations.

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

An optimal controlled *u(t) = [-k,-k] x(t)* is applied to the system. When $x^T(0)=[0,1]$, determine the feedback gain *k* of the system given below that minimizes the following performance index

$$J = \int_0^\infty x^T x \, dt$$

Plot the performance index *J* versus the gain *k*.

Laplace transform table

Item no.	*f(t)*	*F(s)*
1.	$\delta(t)$	1
2.	$u(t)$	$\frac{1}{s}$
3.	$tu(t)$	$\frac{1}{s^2}$
4.	$t^n u(t)$	$\frac{n!}{s^{n+1}}$
5.	$e^{-at}u(t)$	$\frac{1}{s+a}$
6.	$\sin \omega t u(t)$	$\frac{\omega}{s^2+\omega^2}$
7.	$\cos \omega t u(t)$	$\frac{s}{s^2+\omega^2}$

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No. Soalan (1)

Muka surat 1/8

Jangan tulis apa-apa di kedua-dua belah garisan

Q 1

$(M_1 s^2 + Bs + K_1) X_1(s) - Bs X_2(s) = F(s)$

$M_1 \ddot{x}_1 + B\dot{x}_1 + K_1 x_1 - B\dot{x}_2 = f$ — (1)

$(M_2 s^2 + Bs + K_2) X_2(s) - Bs X_1(s) = 0$

$m_2 \ddot{x}_2 + B\dot{x}_2 + K_2 x_2 - B\dot{x}_1 = 0$ — (2)

state variables :- $z_1 = x_1$ $\quad \dot{z}_1 = z_2$ — (a)

$z_2 = \dot{x}_1$

$z_3 = x_2$ $\quad \dot{z}_3 = z_4$ — (b)

$z_4 = \dot{x}_2$

dari (1)

$M_1 \dot{z}_2 + B z_2 + K_1 z_1 - B z_4 = f$

$\dot{z}_2 = -\frac{K_1}{m_1} z_1 - \frac{B}{m_1} z_2 + \frac{B}{m_1} z_4 + \frac{f}{m_1}$ — (c)

dari (2)

$m_2 \dot{z}_4 + B z_4 + K_2 z_3 - B z_2 = 0$

$\dot{z}_4 = \frac{B}{m_2} z_2 - \frac{K_2}{m_2} z_3 - \frac{B}{m_2} z_4$ — (d)

dari (a) → (d)

$$\dot{z} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{K_1}{m_1} & -\frac{B}{m_1} & 0 & \frac{B}{m_1} \\ 0 & 0 & 0 & 1 \\ 0 & \frac{B}{m_2} & -\frac{K_2}{m_2} & -\frac{B}{m_2} \end{bmatrix} z + \begin{bmatrix} 0 \\ \frac{1}{m_1} \\ 0 \\ 0 \end{bmatrix} f$$

$y = [0 \quad 0 \quad 1 \quad 0] z$

* $z_3 = x_2$ //

$y = z_3 = x_2$ //

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No. Soalan 2.

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Muka surat 2/8

Jangan tulis apa-apa di kedua-dua belah garisan

Q2

$$A = \begin{bmatrix} -5 & -1 \\ 3 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 2 \\ 5 \end{bmatrix}, \quad c\,[1 \quad 0].$$

$$[sI - A] = \begin{bmatrix} s+5 & 1 \\ -3 & s+1 \end{bmatrix}$$

a) $\det(sI-A) = (s+1)(s+5) + 3 = s^2 + 6s + 5 + 3 = s^2 + 6s + 8$

$= (s+4)(s+2) \Rightarrow s = -2, -4.$

the system is stable since ~~both of~~ the eigenvalue (poles) are located at the left hand side of the s-plane, hence (−2, −4)

b) $$(sI-A)^{-1} = \frac{\begin{bmatrix} s+1 & -1 \\ 3 & s+5 \end{bmatrix}}{\det(sI-A)} = \frac{\begin{bmatrix} s+1 & -1 \\ 3 & s+5 \end{bmatrix}}{(s+2)(s+4)}$$

$$= \begin{bmatrix} \frac{(s+1)}{(s+2)(s+4)} & \frac{1}{(s+2)(s+4)} \\ \frac{3}{(s+2)(s+4)} & \frac{s+5}{(s+2)(s+4)} \end{bmatrix} = \begin{bmatrix} \frac{A}{s+2} + \frac{B}{s+4} & \frac{C}{s+2} + \frac{D}{s+4} \\ \frac{E}{s+2} + \frac{F}{s+4} & \frac{G}{s+2} + \frac{H}{s+4} \end{bmatrix}$$

$A(s+4) + B(s+2) = s+1$ $\qquad C(s+4) + D(s+2) = -1$

$s=-2, \quad s=-4$ $\qquad s=-2 \quad s=-4$

$2A = -1, \quad -2B = -3$ $\qquad 2C = -1 \quad -2D = -1$

$A = -\frac{1}{2} \quad B = \frac{3}{2}$ $\qquad C = -\frac{1}{2} \quad D = +\frac{1}{2}.$

$E(s+4) + F(s+2) = 3$ $\qquad G(s+4) + H(s+2) = s+5$

$s=-2 \quad s=-4$ $\qquad s=-2 \quad s=-4$

$2E = 3 \quad -2F = -3$ $\qquad 2G = 3 \quad -2H = 1$

$E = \frac{3}{2} \quad F = \frac{-3}{2}$ $\qquad G = \frac{3}{2} \quad H = -\frac{1}{2}.$

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No. Soalan 2

Muka surat 3/8

Jangan tulis apa-apa di kedua-dua belah garisan

Q2 (b)

$$(sI-A)^{-1} = \begin{bmatrix} \left(\frac{-\frac{1}{2}}{s+2} + \frac{\frac{3}{2}}{s+4}\right) & \left(\frac{-\frac{1}{2}}{s+2} + \frac{\frac{1}{2}}{s+4}\right) \\ \left(\frac{\frac{3}{2}}{s+2} - \frac{\frac{3}{2}}{s+4}\right) & \left(\frac{\frac{3}{2}}{s+2} - \frac{\frac{1}{2}}{s+4}\right) \end{bmatrix} \quad -(1)$$

$$\phi(t) = \mathcal{L}^{-1}\left[(sI-A)^{-1}\right]$$

$$= \begin{bmatrix} -\frac{1}{2}e^{-2t} + \frac{3}{2}e^{-4t} & -\frac{1}{2}e^{-2t} + \frac{1}{2}e^{-4t} \\ \frac{3}{2}e^{-2t} - \frac{3}{2}e^{-4t} & \frac{3}{2}e^{-2t} - \frac{1}{2}e^{-4t} \end{bmatrix} \quad -(2)$$

(c) $u(t) = e^{-t} \Rightarrow u(s) = \frac{1}{s+1}$.

$\dot{x} = Ax + Bu.$

$sX(s) - x(0) = AX(s) + Bu(s)$, $x(0)=0$.

$(sI-A)X(s) = Bu(s)$

$X(s) = (sI-A)^{-1}Bu(s). \quad - (3).$

$$Bu(s) = \begin{bmatrix} 2 \\ 5 \end{bmatrix}\begin{bmatrix} \frac{1}{s+1} \end{bmatrix} = \begin{bmatrix} \frac{2}{s+1} \\ \frac{5}{s+1} \end{bmatrix}.$$

dari (1).

$$X(s) = \begin{bmatrix} \left(\frac{-\frac{1}{2}}{s+2} + \frac{\frac{3}{2}}{s+4}\right) & \left(\frac{-\frac{1}{2}}{s+2} + \frac{\frac{1}{2}}{s+4}\right) \\ \left(\frac{\frac{3}{2}}{s+2} - \frac{\frac{3}{2}}{s+4}\right) & \left(\frac{\frac{3}{2}}{s+2} - \frac{\frac{1}{2}}{s+4}\right) \end{bmatrix}\begin{bmatrix} \frac{2}{s+1} \\ \frac{5}{s+1} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-1}{(s+1)(s+2)} + \frac{3}{(s+1)(s+4)} + \frac{-\frac{5}{2}}{(s+1)(s+2)} + \frac{\frac{5}{2}}{(s+1)(s+4)} \\ \frac{3}{(s+1)(s+2)} - \frac{3}{(s+1)(s+4)} + \frac{\frac{15}{2}}{(s+1)(s+2)} - \frac{\frac{5}{2}}{(s+1)(s+4)} \end{bmatrix}$$

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No. Soalan 2

Muka surat 4/8

Jangan tulis apa-apa di kedua-dua belah garisan

Q2 (c)

$$X(s) = \begin{bmatrix} \dfrac{-\frac{7}{2}}{(s+1)(s+2)} + \dfrac{\frac{11}{2}}{(s+1)(s+4)} \\[2ex] \dfrac{\frac{21}{2}}{(s+1)(s+2)} - \dfrac{\frac{11}{2}}{(s+1)(s+4)} \end{bmatrix} = \begin{bmatrix} \dfrac{A}{s+1} + \dfrac{B}{s+2} + \dfrac{C}{s+1} + \dfrac{D}{s+4} \\[2ex] \dfrac{E}{s+1} + \dfrac{F}{s+2} + \dfrac{G}{s+1} + \dfrac{H}{s+4} \end{bmatrix}$$

$A(s+2) + B(s+1) = -\frac{7}{2}$ $\qquad$ $C(s+4) + D(s+1) = \frac{11}{2}$

$s=-1$	$s=-2$	$s=-1$	$s=-4$
$A = -\frac{7}{2}$	$-B = -\frac{7}{2}$	$3C = \frac{11}{2}$	$-3D = +\frac{11}{2}$
$A = -\frac{7}{2}$	$B = \frac{7}{2}$	$C = \frac{11}{6}$	$D = -\frac{11}{6}$

$E(s+2) + F(s+1) = \frac{21}{2}$ $\qquad$ $G(s+4) + H(s+1) = -\frac{11}{2}$

$s=-1$	$s=-2$	$s=-1$	$s=-4$
$E = \frac{21}{2}$	$-F = \frac{21}{2}$	$3G = -\frac{11}{2}$	$-3H = -\frac{11}{2}$
	$F = -\frac{21}{2}$	$G = -\frac{11}{6}$	$H = \frac{11}{6}$

$$X(s) = \begin{bmatrix} \dfrac{-\frac{7}{2}}{s+1} + \dfrac{\frac{7}{2}}{s+2} + \dfrac{\frac{11}{6}}{s+1} - \dfrac{\frac{11}{6}}{s+4} \\[2ex] \dfrac{\frac{21}{2}}{s+1} - \dfrac{\frac{21}{2}}{s+2} - \dfrac{\frac{11}{6}}{s+1} + \dfrac{\frac{11}{6}}{s+4} \end{bmatrix} = \begin{bmatrix} \dfrac{-\frac{5}{3}}{s+1} + \dfrac{\frac{7}{2}}{s+2} - \dfrac{\frac{11}{6}}{s+4} \\[2ex] \dfrac{\frac{26}{3}}{s+1} - \dfrac{\frac{21}{2}}{s+2} + \dfrac{\frac{11}{6}}{s+4} \end{bmatrix}$$

$$x(t) = \begin{bmatrix} -\frac{5}{3}e^{-t} + \frac{7}{2}e^{-2t} - \frac{11}{6}e^{-4t} \\[2ex] \frac{26}{3}e^{-t} - \frac{21}{2}e^{-2t} + \frac{11}{6}e^{-4t} \end{bmatrix}$$

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No. Soalan 4.

Muka surat 5/8

Jangan tulis apa-apa di kedua-dua belah garisan

Q4 a)

$$A = \begin{bmatrix} 0 & 1 \\ 20.6 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad C = [1 \quad 0]$$

$$BK = \begin{bmatrix} 0 \\ 1 \end{bmatrix} [K_1 \quad K_2] = \begin{bmatrix} 0 & 0 \\ K_1 & K_2 \end{bmatrix}.$$

poles $= -1.8 \pm j2.4$.

$$(s + 1.8 + j2.4)(s + 1.8 - j2.4)$$

$$= s^2 + 1.8s - \cancel{j2.4s} + 1.8s + 3.24 - \cancel{j4.32} + \cancel{j2.4s} + \cancel{j4.32} + 5.76.$$

$$= s^2 + 3.6s + 9 \Rightarrow \text{desired CE.} \quad -(1)$$

But $A - BK = \begin{bmatrix} 0 & 1 \\ 20.6 - K_1 & -K_2 \end{bmatrix}$

$$[sI - (A - BK)] = \begin{bmatrix} s & -1 \\ (K_1 - 20.6) & s + K_2 \end{bmatrix}$$

$$\det[sI - (A - BK)] = s(s + K_2) + K_1 - 20.6$$

$$= s^2 + K_2 s + K_1 - 20.6 \quad -(2).$$

bandingkan (1) dg (2).

$$\Rightarrow K_2 = 3.6 \qquad K_1 - 20.6 = 9$$

$$K_1 = 29.6$$

$$\Rightarrow A - BK = \begin{bmatrix} 0 & 1 \\ -9 & -3.6 \end{bmatrix}$$

$$K = [29.6 \quad 3.6]$$

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No. Soalan 4

Muka surat 6/8

Jangan tulis apa-apa di kedua-dua belah garisan

Q4 b)

eigenvalues = -8 ⇒ cl. poles.

$(s+8)(s+8) = s^2+16s+64$ ⇒ desired CE — (1).

since the ss is simple, transformation ✗.

$$A - LC = \begin{bmatrix} 0 & 1 \\ 20.6 & 0 \end{bmatrix} - \begin{bmatrix} L_1 \\ L_2 \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \\ 20.6 & 0 \end{bmatrix} - \begin{bmatrix} L_1 & 0 \\ L_2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -L_1 & 1 \\ (20.6 - L_2) & 0 \end{bmatrix}$$

$$sI - (A-LC) = \begin{bmatrix} s+L_1 & -1 \\ L_2 - 20.6 & s \end{bmatrix}$$

$$\det[sI - (A-LC)] = s(s+L_1) + L_2 - 20.6$$
$$= s^2 + L_1 s + L_2 - 20.6 \quad -(2)$$

bandingkan (1) & (2).

$L_1 = 16$ // , $L_2 - 20.6 = 64$

$L_2 = 84.6$ //

$$L = \begin{bmatrix} 16 \\ 84.6 \end{bmatrix}$$

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No. Soalan 5

Muka surat 7/8

Jangan tulis apa-apa di kedua-dua belah garisan

Q5

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}u(t)$$

$$u(t) = (-k \quad -k)\begin{bmatrix}x_1\\ x_2\end{bmatrix} = -kx_1 - kx_2.$$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ -k & -k\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

$$\dot{x}\ \cancel{D} = Dx$$

$$D = \begin{bmatrix}0 & 1\\ -k & -k\end{bmatrix}$$

$$D^T P + PD = -I$$

$$\cancel{\begin{bmatrix}0 & -k\\ 1 & -k\end{bmatrix}\begin{bmatrix}p_{11} & p_{12}\\ p_{12} & p_{22}\end{bmatrix} - \begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}}$$

$$\begin{bmatrix}0 & -k\\ 1 & -k\end{bmatrix}\begin{bmatrix}p_{11} & p_{12}\\ p_{12} & p_{22}\end{bmatrix} + \begin{bmatrix}p_{11} & p_{12}\\ p_{12} & p_{22}\end{bmatrix}\begin{bmatrix}0 & 1\\ -k & -k\end{bmatrix} = \begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}$$

$$\begin{bmatrix}-kp_{12} & -kp_{22}\\ p_{11}-kp_{12} & p_{12}-kp_{22}\end{bmatrix} + \begin{bmatrix}-kp_{12} & p_{11}-kp_{12}\\ -kp_{22} & p_{12}-kp_{22}\end{bmatrix} = \begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}$$

$$-2kp_{12} = -1$$

$$p_{12} = \frac{1}{2k}$$

$$p_{12} - kp_{22} + p_{12} - kp_{22} = -1$$

$$2p_{12} - 2kp_{22} = -1$$

$$\cancel{\frac{2}{2k} = 2kp_{22} - 1}$$

$$2k\,p_{22} = \frac{1}{k} + 1$$

$$p_{22} = \frac{1}{2k^2} + \frac{1}{2k} = \frac{k+1}{2k^2}$$

$$-kp_{22} + p_{11} - kp_{12} = 0$$

$$-\frac{(k+1)}{2k} + p_{11} - \frac{1}{2} = 0$$

$$p_{11} = \frac{1}{2} + \frac{(k+1)}{2k}$$

$$= 2k+1$$

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No. Soalan 5.

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Muka surat 8/8.

Jangan tulis apa-apa di kedua-dua belah garisan

Q5

$$J = \int_0^\infty x^T x \, dt = x^T(0) \, P \, x(0)$$

$$= \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} p_{12} \\ p_{22} \end{bmatrix}$$

$$= p_{22}$$

$$J = p_{22} = \frac{k+1}{2k^2}$$

~~$\frac{dJ}{dk} = \frac{2k^2(1) - (k+1)(4k)}{4k^4}$~~

~~J_{min} when $\frac{dJ}{dk} = 0$~~

~~$2k^2 - 4k^2 - 4k = 0$~~

~~$-2k^2 - 4k = 0$~~

~~$2k^2 + 4k = 0$~~

~~$2k(k+2) = 0$~~

~~$k = 0, -2.$~~

*anggap k -ve tidak diterima.

J_{min} bila $k \to \infty$

tetapi $k\ \infty$ adalah tidak mungkin

so. let $k = 50$.

$J = 0.0102$

k	J
5	0.12
10	0.055
15	0.036
20	0.026
25	0.021

k	J
30	0.017
35	0.015
40	0.013
45	0.011
50	0.010