WORKSHOP
ON SYSTEM IDENTIFICATIONS
FOR SENSING AND PROCESS
(SYSID 2008)

CHAPTER 3

ARX MODELLING
AND
CONTROLLER DESIGN

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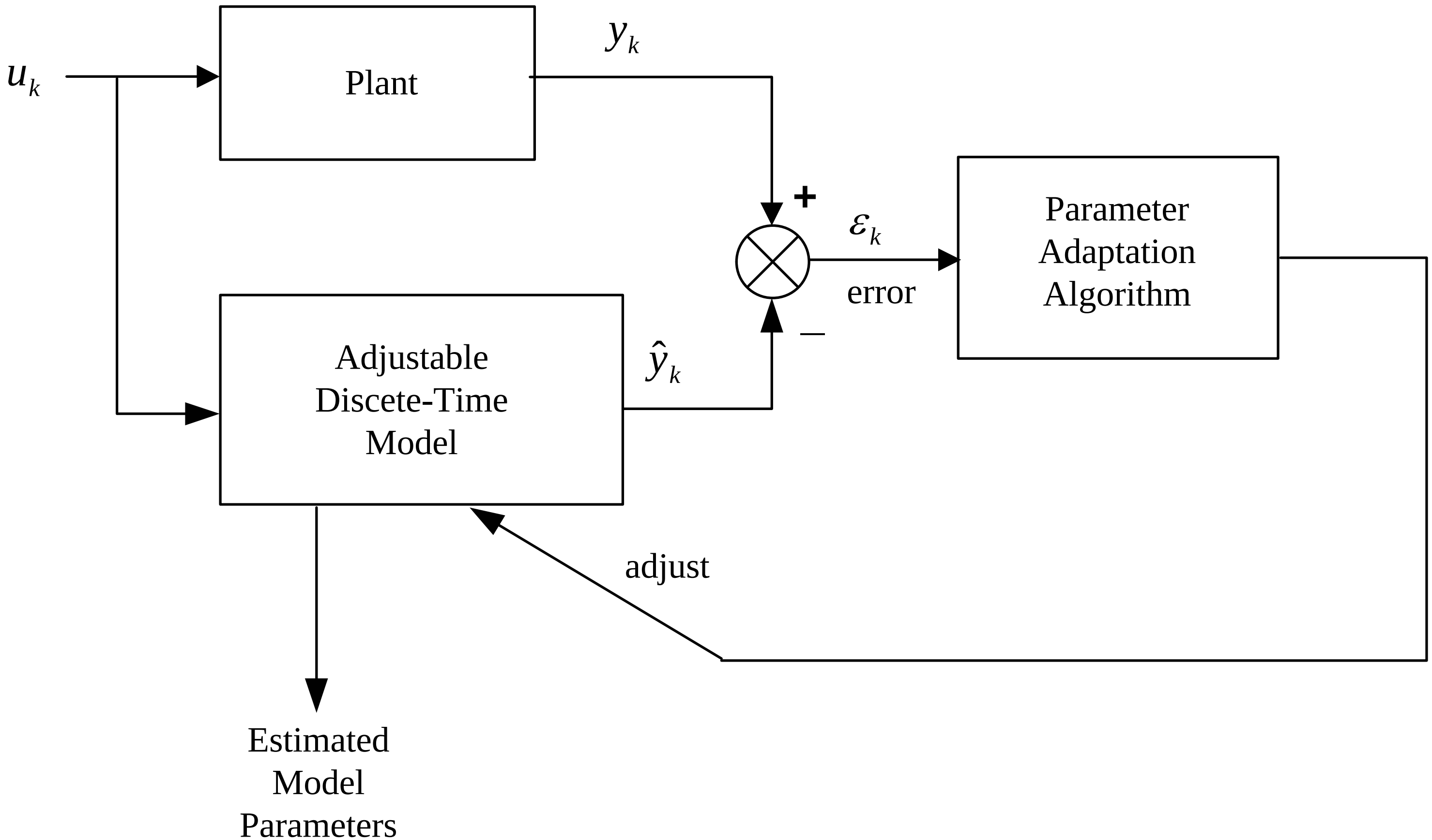
OUTLINE

- SOME THEORIES ON SYSTEM IDENTIFICATION
- MODEL IDENTIFICATION USING MATLAB
- FEEDBACK CONTROL SYSTEM DESIGN
- SIMULATION USING SIMULINK

SYSTEM IDENTIFICATION

- Deriving mathematical model
  - From measured input-output data of a certain system dynamic.
- Dynamic model
  - Required for design and implementing high performance control system.
- Obtained model
  - Needs to be validated to verify its preciseness before can be used.
- Validation
  - Comparing the output of the obtained model and the output of the plant
  - Using part of the experimental data that has been reserved for this purpose.



MODEL IDENTIFICATION TECHNIQUE

❑ Parametric Model Structure

- represented by difference equation

$$y(k)+a_1 y(k-1)+\cdots\cdots+a_{na}y(k-na)=b_1u(k-d)+b_2u(k-d-1)$$
$$+\cdots\cdots+b_{nb}u(k-d-nb+1)+e(k)$$

$$A(z^{-1})y(k)=B(z^{-1})u(k-d)+e(k)$$

$$A(z^{-1})=1+a_1z^{-1}+\cdots\cdots+a_{na}z^{-na}$$

$$B(z^{-1})=b_1+b_2z^{-1}+\cdots\cdots+b_{nb}z^{-nb+1}$$

d : time delay
na : no. of poles
nb : no. of zeros
u(k) : input
y(k) : output
e(k) : noise

PARAMETRIC MODEL STRUCTURE

- Few structures (Ljung 2002)
  - Auto-Regressive with Exogenous Input (ARX) model
  - Auto-Regressive Moving Average with Exogenous Input (ARMAX) model
  - Output-Error (OE) model
  - Box-Jenkins (BJ) model
  - Other models
- ARX model
  - basis model in forming few other model structures

❑ ARMAX Model

- represented by equation

$$A(z^{-1})y(k) = B(z^{-1})u(k-d) + C(z^{-1})e(k)$$

$$C(z^{-1}) = 1 + c_1 z^{-1} + \cdots + c_{nc} z^{-nc}$$

❑ OE Model

- represented by equation

$$y(k) = \frac{B(z^{-1})}{F(z^{-1})}u(k-d) + e(k)$$

$$F(z^{-1}) = 1 + f_1 z^{-1} + \cdots + f_{nf} z^{-nf}$$

❑ BJ Model

- represented by equation

$$y(k)=\frac{B(z^{-1})}{F(z^{-1})}u(k-d)+\frac{C(z^{-1})}{D(z^{-1})}e(k)$$

$$D(z^{-1})=1+d_1z^{-1}+\cdots+d_{nd}z^{-nd}$$

❑ General parametric model

- represented by equation

$$A(z^{-1})y(k)=\frac{B(z^{-1})}{F(z^{-1})}u(k-d)+\frac{C(z^{-1})}{D(z^{-1})}e(k)$$

GENERATING INPUT SIGNAL

- Good parameters identification requires the usage of input signal that are rich in frequencies.

- Two methods

a) Pseudo-Random Binary Sequences (PRBS)

- Generating sequences of different width square pulses
- Implementation a little bit difficult.

b) Using sinusoidal signals

$$u(k) = \sum_{i=1}^{p} a_i \cos \omega_i t_s k$$

a_i : amplitude

ω_i : frequency

t_s : *sampling time (sec)*

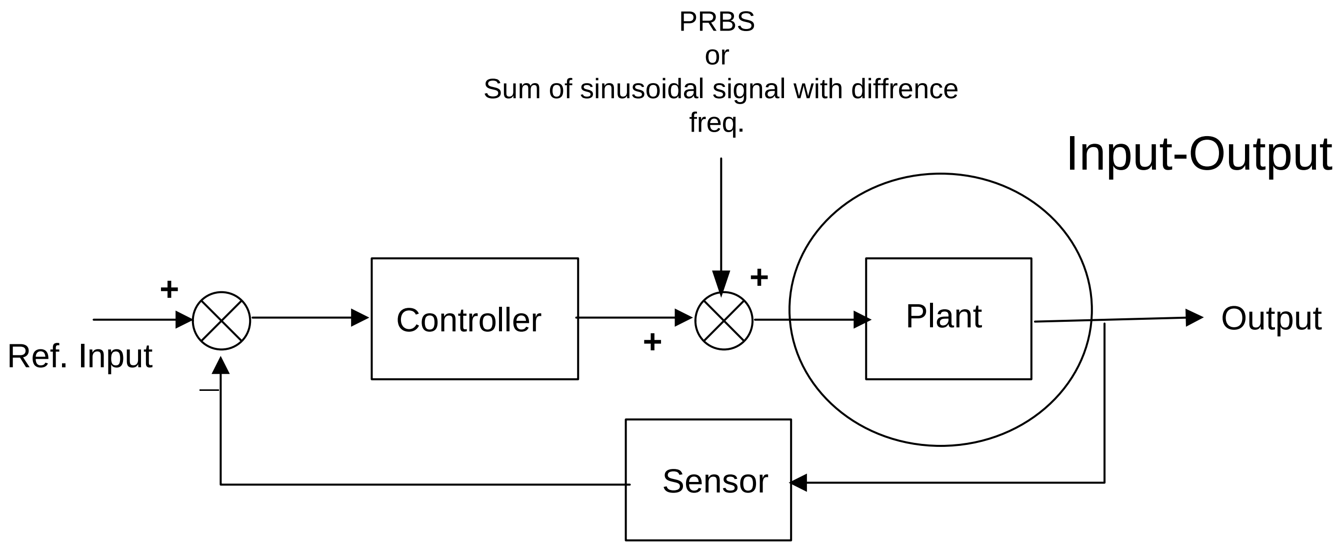
$P = n/2$

n : the number of model parameter that needs to be identified

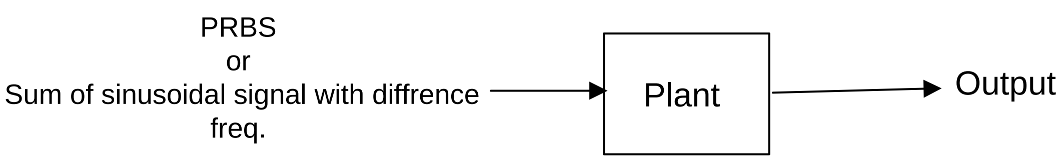
- If $P = 3$
  - A maximum of six model parameters can be obtained.
  - Generated input signal

$$u(k) = a_1 \cos \omega_1 t_s k + a_2 \cos \omega_2 t_s k + a_3 \cos \omega_3 t_s k$$

Data Acquisition



Closed-Loop



Open-Loop

PLANT MODEL

- Derived from the measured input and output signals of a real plant that needs to be identified.

- The ARX parametric model structure
  - Simple structure
  - Represented by a simple linear difference equation.

Assuming noise is zero,

$$y(k) + a_1 y(k-1) + \cdots\cdots + a_{na} y(k-na) = b_1 u(k-d) + b_2 u(k-d-1) + \cdots\cdots + b_{nb} u(k-d-nb+1)$$

$$na \geq nb$$

$$\frac{Y(z^{-1})}{U(z^{-1})} = z^{-d}\,\frac{B(z^{-1})}{A(z^{-1})}$$

- ❑ Minimum phase model
  - •Can be obtained using large value of sampling time

- ❑ Non-minimum phase model
  - •Can be obtained using small value sampling of time

(Astrom & Wittenmark 1997, Soderstrom & Stoica 1989).

PARAMETRIC MODEL IDENTIFICATION ALGORITHM

- Parameters approximation can be obtained using the recursive least square (RLS) algorithm.

- The equations involved: Regression Vector:

Regression Vector:

$$\psi_k^{\ T} = [-y_{k-1}, \cdots, -y_{k-na}, u_{k-1}, \cdots, u_{k-nb}] = [\psi_1, \cdots, \psi_{na+nb}]$$

Approximate Parameters Vector:

$$\hat{\theta}_k^{\ T} = [\hat{a}_1, \cdots, \hat{a}_{\mathrm{na}}, \hat{b}_1, \cdots, \hat{b}_{nb}] = [\hat{\theta}_1, \cdots, \hat{\theta}_{na+nb}]$$

Predicted Error:

$$e_{k+1} = y_{k+1} - \hat{y}_k$$

$$\hat{y}_k = \psi_{k+1}{}^T \hat{\theta}_k$$

Update Approximation:

$$\hat{\theta}_{k+1} = \hat{\theta}_k + K_k e_{k+1}$$

Correcting Vector:

$$K_k = \frac{P_k \psi_{k+1}}{\psi_{k+1}{}^T P_k \psi_{k+1} + \lambda}$$

Pk : Covariant Matrix

λ *: Forgetting Factor*

- Effects of errors due to truncation and round-off during computation of covariance matrix (P_k)
  - P_k can lost its symmetry
  - positive definiteness
  - divergence

- U-D factorisation method for computing P_k (Isermann et al. 1992, Zhang & Li 1999)
  - Fast convergence
  - Effectiveness
  - Symmetry of P_k is guaranteed

- Modification to the correcting vector with reference to U-D factorisation method

Correcting Vector Using U-D factorization Algorithm:

$$K_k = \frac{U_k g_k}{\alpha_k}$$

$$f_k = U_k^{\ T} \psi_{k+1}$$

$$g_k = D_k f_k$$

$$\alpha_k = f_k^{\ T} g_k + \lambda$$

U_k : Upper Triangular Matrix
D_k : Diagonal Matrix
λ : Forgetting Factor

MODAL VALIDATION

- Using a part of experimental data that was not used

- Acceptance or rejection of obtained model based on criteria:

  - Akaike's Final Prediction Error
    (Ljung 1999,2002; Soderstrom & Stoica 1989)
  - Akaike's Information Criteria
    (Ljung 1999,2002; Soderstrom & Stoica 1989)
  - Best Fit (Ljung 2002)
  - Other

- These criterions show the preciseness of the approximate model as compared to the true model.

- Akaike's Final Prediction Error:

$$FPE = V \cdot \frac{(1 + d/N)}{(1 - d/N)}$$

V : Loss Function (Ljung 1999; Soderstrom & Stoica 1989)
d : No. of approximated parameters
N : No. of samples

$$V = \frac{e^2(k)}{N} = \frac{e^T(k).e(k)}{N}$$

e(*k*) : error vector

$$e(k) = \begin{bmatrix} e_k & e_{k-1} & \cdots & e_{k-N} \end{bmatrix}^T$$

Selection of model from various order is based on the smallest value of FPE.

- Akaike's Information Criteria:

$$AIC = \log(V \cdot (1 + 2d/N))$$

Selection of model from various order is based on the smallest value of AIC.

- Best fitting criteria:

$$fit = 100 \cdot \left[1 - \frac{norm(\hat{y} - y)}{norm(y - \bar{y})} \right] \%$$

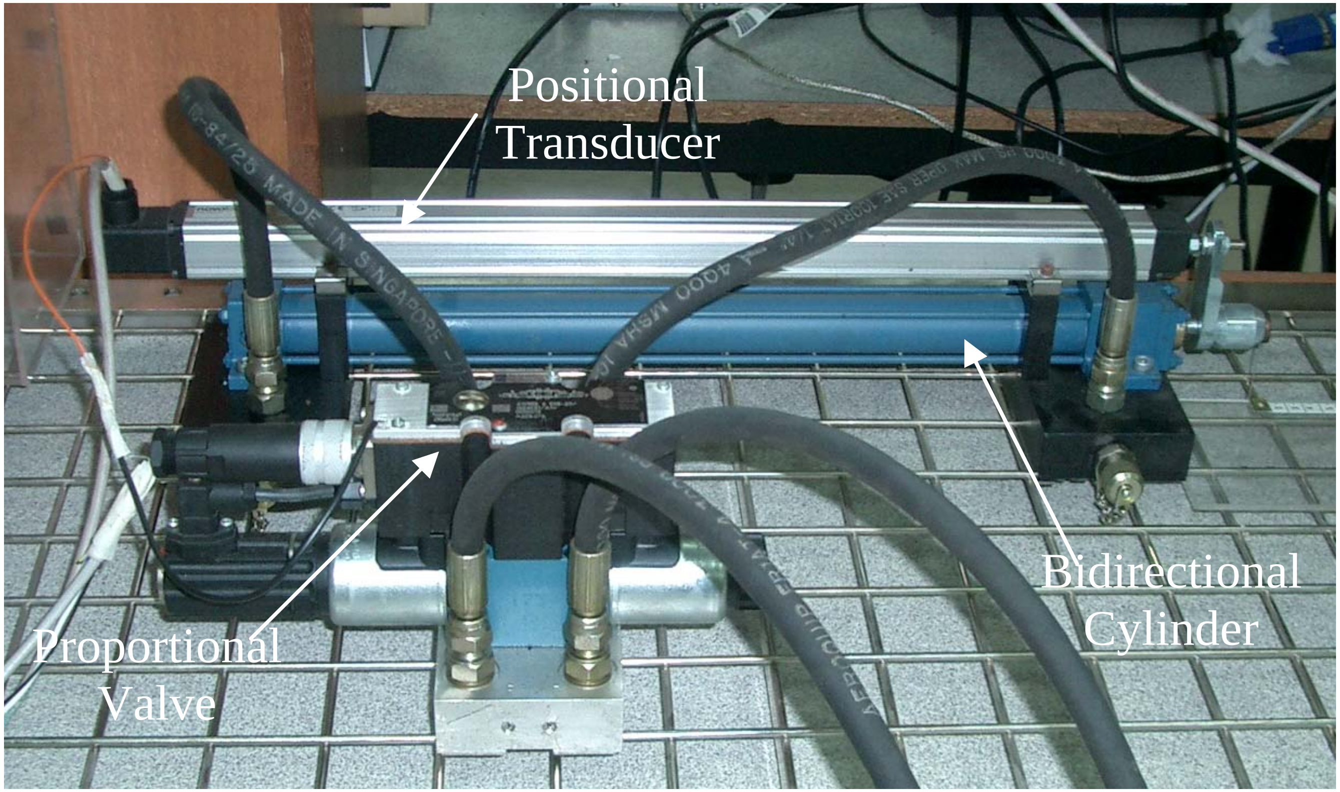
y : True value

$\hat{y}$: Approximate value

$\bar{y}$: Mean value

Model selection will be based on the highest percentage value.

Plant



Industrial standard hydraulic actuator

PLANT DESCRIPTION

- Industrial standard hydraulic actuator
- A bidirectional cylinder type (brand name: Novotechnik)
- The piston diameter is 25mm, piston rod diameter 16mm, stroke 400mm and piston area ratio 1.6:1.
- Inductive type built-in position transducer.
- Pressurised fluid flow is control by electronic control valve (band name: Rexroth)
- Control valve is of proportional and directional type.
- Valve input voltage: ±10V dc and current range (4-20) mA.
- Pressurised pump (Brand name: Bosch Rexroth) can regulate fluid source of up to 2300 psi or 160 bar.

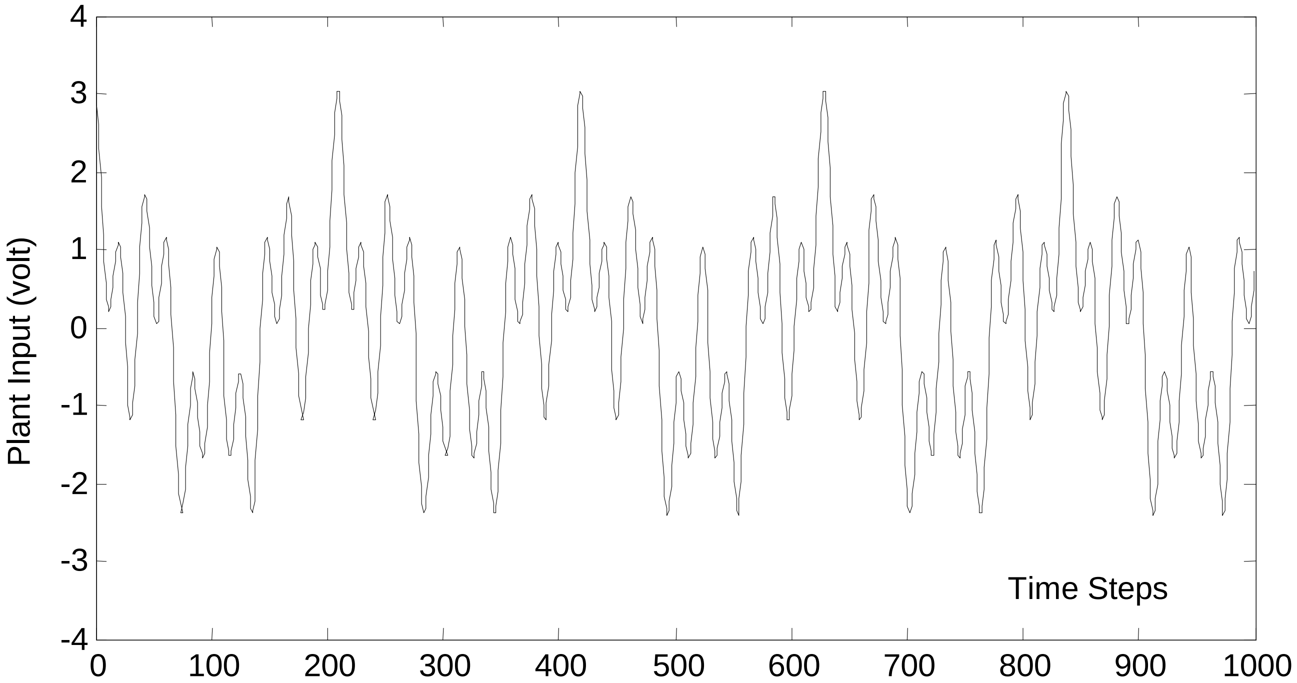
CASE STUDIES:

- Model Identification
  - Input signal to the plant - generated using three different frequencies

$$V_{in}(k) = cos\, 0.5 t_s k + cos\, 2 t_s k + cos\, 5 t_s k$$

t_s : sampling time

- Using three different frequencies for the input signal.
  - Models order obtain limited to second and third order
  - Higher order models may produce inaccurate model or unstable output.
  - Third order model will represent the nearest model of true plant.



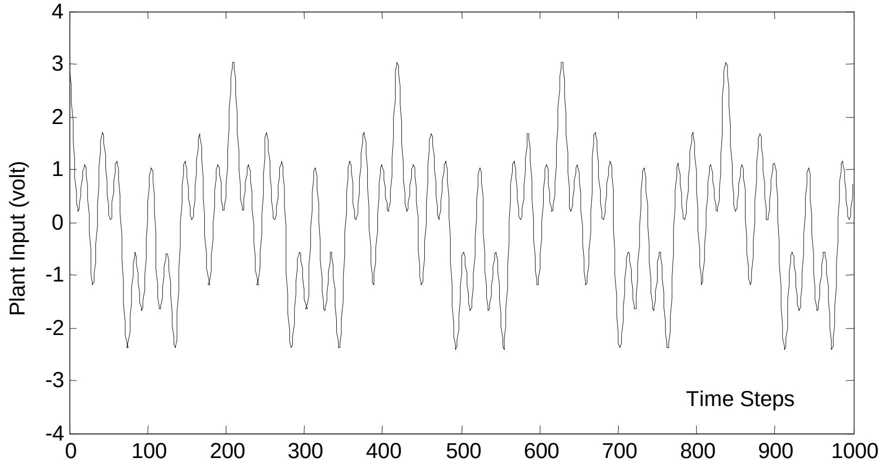
Input signal generated

INPUT-OUTPUT DATA

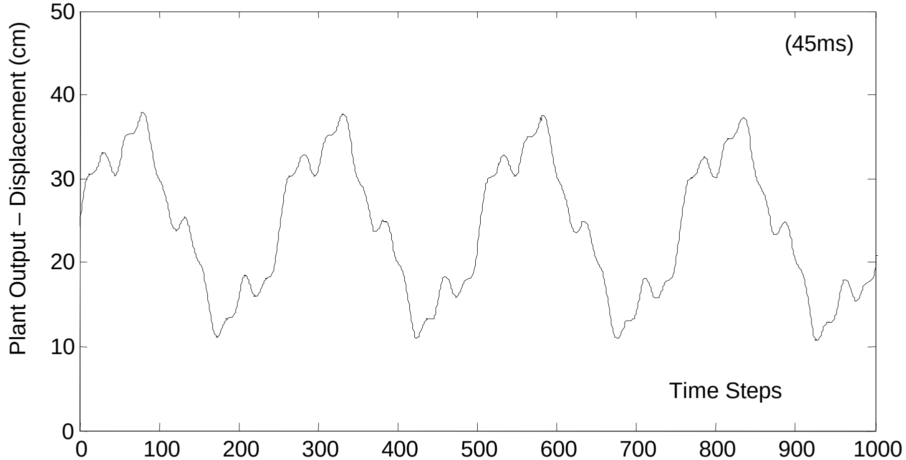
- ❑ Input-output data inside the CD ROM
- ❑ Folder: ARX_Modelling:
  - M-File: MODELA.m
    Sampling time = 45ms

PLANT MODEL

- Model A is derive from m-file MODELA.m



Input Signal



Output Signal

- The input-output signal of the plant.
- Divided into 2 parts
  - 1 - 500 samples
  - 501 - 1000 samples.
- First part
  - 501 - 1000 samples
  - Getting model
- Second part
  - 501 - 1000 samples
  - Model validation

MODEL IDENTIFICATION

- Using Matlab System Identification Toolbox

- First part of input-output signal
  - Plant model ARX331

$$\frac{B_o(z^{-1})}{A_o(z^{-1})} = \frac{0.1478z^{-1} + 0.1617z^{-2} - 0.06549z^{-3}}{1 - 0.9139z^{-1} - 0.2599z^{-2} + 0.1743z^{-3}}$$

Simplified:

$$\frac{B_o(z^{-1})}{A_o(z^{-1})} = \frac{0.1478z^{-1}(1 + 1.0940z^{-1} - 0.4431z^{-2})}{1 - 0.9139z^{-1} - 0.2599z^{-2} + 0.1743z^{-3}}$$

Time delay, $d = 1$

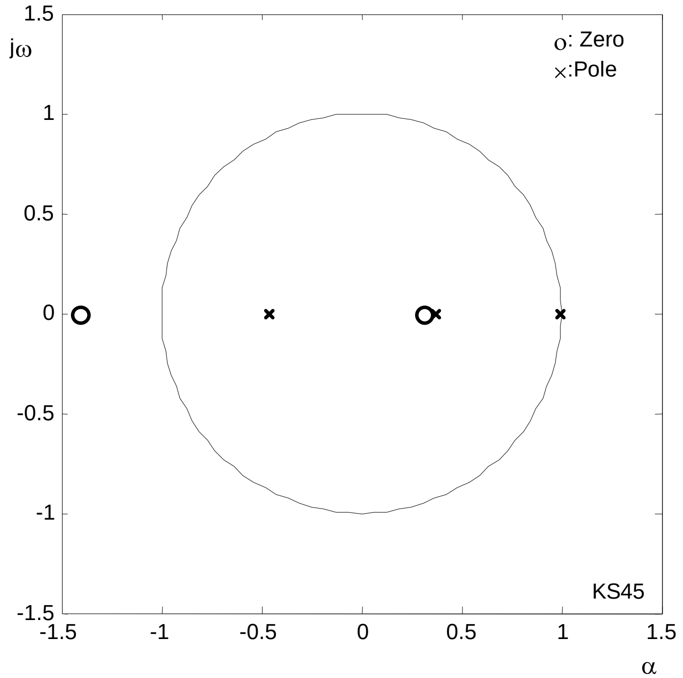
Zero polynomials:

$$B_c(z^{-1}) = 1 + 1.0940z^{-1} - 0.4431z^{-2}$$

Factorised:

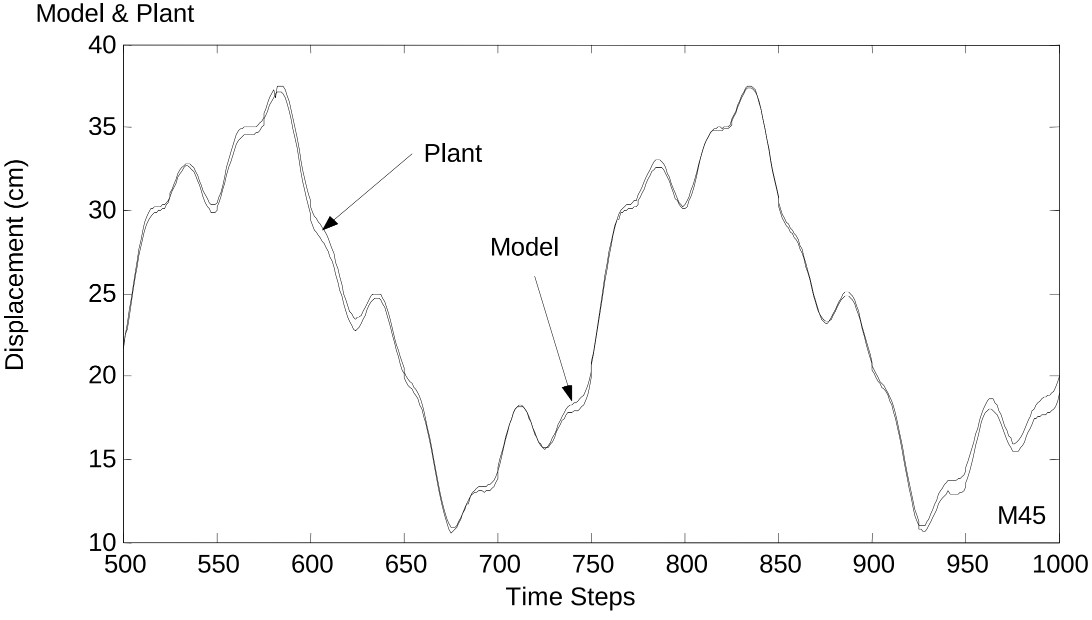
$$z = -1.4086$$
$$z = 0.3146$$

- Obtained Model:
  - Non-minimum phase system



Pole-zero plot

- Second part of input-output signals
  - Model validation
  - Input signal - input to the obtain model
  - Output signal - compared with the output of the obtained model
- Result



Model and plant output signals

- Using Model Selection Criterion :
  - Best Fit: 94.6 %
  - Loss Function: 0.0022
  - Final Prediction Error: 0.0023
  - Akaike's Information Criteria: –6.094

- Model can be accepted
  - Based on the smallest values criteria of FPE and AIC
  - Good percentage best fit