System Identification & Parameter Estimation

Wb2301: SIPE lecture 5

Open & closed loop, SISO & MIMO systems

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Resume previous lecture: FRF estimation with random data

- Estimator for open loop FRF, coherence

$$\hat{H}_{uy}(f) = \frac{\hat{S}_{uy}(f)}{\hat{S}_{uu}(f)}; \quad \hat{S}_{uy}(f) = \frac{1}{N} U^*(f) Y(f)$$

$$\hat{\gamma}_{uy}{}^2(f) = \frac{\left|\hat{S}_{uy}(f)\right|^2}{\hat{S}_{uu}(f)\hat{S}_{yy}(f)}$$

- Smoothed spectral densities
  - Welch

$$\hat{S}_{uu}(f) = \frac{1}{D}\sum_{d=1}^{D} S_{uu}(f) = \frac{1}{DN_D}\sum_{d=1}^{D} U(-f)U(f)$$

  - Frequency averaging

$$\hat{S}_{uu}(f_c) = \frac{1}{D}\sum_{d=1}^{D} \hat{S}_{uu}(f_d); \quad f_c = \frac{1}{D}\sum_{d=1}^{D} f_d$$

Variance of FRF estimator, with random data

- Variance FRF estimator

$$\frac{\sigma_{|\hat{H}|}^{2}(f)}{|H(f)|^{2}} = \sigma_{\angle\hat{H}}^{2}(f) = \frac{1}{2D}\left(\frac{1-\gamma_{uy}^{2}(f)}{\gamma_{uy}^{2}(f)}\right)$$

$$\sigma_{|\hat{H}|}(f) = \frac{\sqrt{1-\hat{\gamma}_{uy}^{2}(f)}}{|\hat{\gamma}_{uy}(f)|\sqrt{2D}}|\hat{H}(f)|$$

$$\sigma_{\angle\hat{H}}(f) = \frac{\sqrt{1-\hat{\gamma}_{uy}^{2}(f)}}{|\hat{\gamma}_{uy}(f)|\sqrt{2D}}$$

Variance decreases with

-Averaging

-Coherence

* Welstead PE, Automatica, 1981

* Bendat & Piersol, 2000

FRF measurements with multiple periods of a periodic signal

- Periodic signal: N samples per period, M periods

- Sample mean (in freq domain)

$$\hat{U}(f) = \frac{1}{M}\sum_{l=1}^{M} U^{[l]}(f)$$

$$\hat{S}_{UU}(f) = \frac{1}{N}\hat{U}^*(f)\hat{U}(f) = \frac{1}{N}\left|\hat{U}(f)\right|^2$$

- Sample (co-)variance (in freq domain)

$$\sigma_{\hat{U}}{}^2(f) = \frac{1}{N(M-1)}\sum_{l=1}^{M}\left|U^{[l]}(f) - \hat{U}(f)\right|^2$$

$$\sigma_{\hat{U}\hat{Y}}{}^2(f) = \frac{1}{N(M-1)}\sum_{l=1}^{M}\left(U^{[l]}(f) - \hat{U}(f)\right)^*\left(Y^{[l]}(f) - \hat{Y}(f)\right)$$

FRF measurements with multiple periods of a periodic signal

- Periodic signal
  - N samples per period
  - M periods

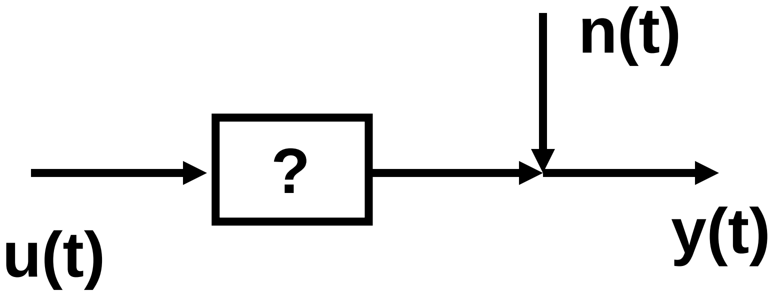
- Frequency response function (FRF)

$$\hat{H}(f) = \frac{\hat{Y}(f)}{\hat{U}(f)}$$

- Variance FRF

$$\sigma_{\hat{H}}^{2}(f) = \frac{1}{M}\left|\hat{H}(f)\right|^{2}\left(\frac{\sigma_{\hat{Y}}^{2}(f)}{\hat{S}_{YY}(f)} + \frac{\sigma_{\hat{U}}^{2}(f)}{\hat{S}_{UU}(f)} - 2re\left(\frac{\sigma_{\hat{U}\hat{Y}}^{2}(f)}{\hat{S}_{UY}(f)}\right)\right)$$

FRF measurements with multiple periods of a periodic signal



$$\hat{H}(f) = \frac{\hat{Y}(f)}{\hat{U}(f)}$$

- Particular case: only output noise, no input noise
- Variance FRF

$$\sigma_H{}^2(f) = \frac{1}{M}\left|H(f)\right|^2 \left(\frac{\sigma_Y{}^2(f)}{S_{YY}(f)} \right)$$

Contents Lecture 5

- Open-loop and closed loop systems:

- Multi-input multi-output (MIMO) systems
  - Multivariable system identification
  - MIMO frequency response function
  - Ordinary coherence : one output - one input
  - Multiple coherence: is an output linearly related with the inputs
  - Partial coherence: is one output linearly related with one input

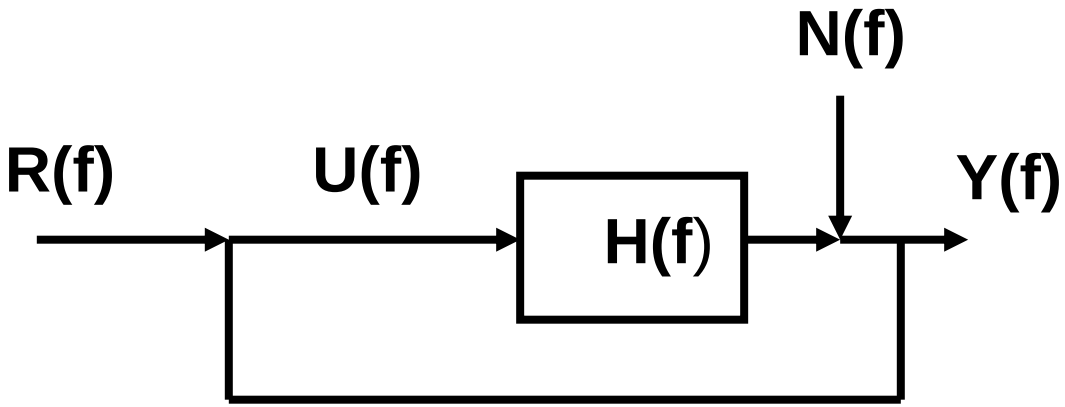
Basic theory: causality

- Physical systems are causal: output depends on *previous* values of the input.
- Anti-causal: depends on *only future* values of the input.
=> input-output are exchanged
- Non-causal: depends on previous and future values of the input.
=> closed-loop systems

Examples of closed loop identification

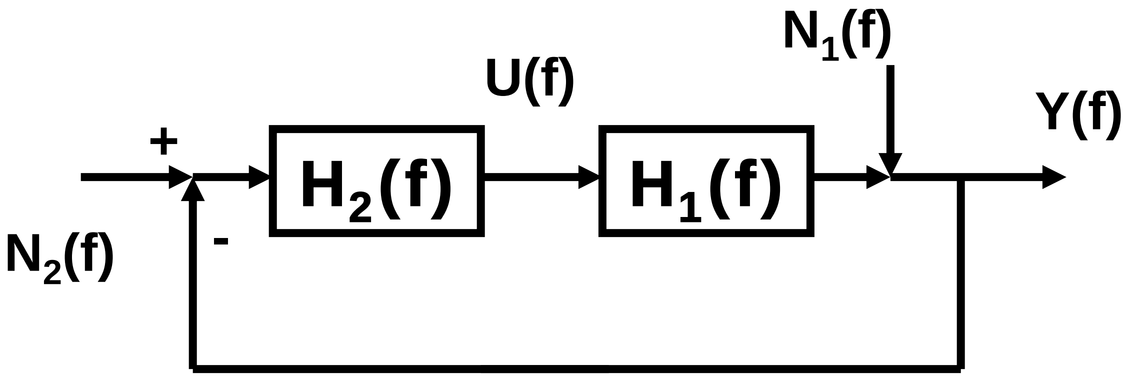
- Human-machine interaction (driving, steering, etc.)
  - Interaction between two systems!
- Human motion control
  - Muscle force depends on activation, activation depends on reflexes, reflexes depend on movement.
- Chemical/nuclear plants
  - Plant is unstable, so a controller is needed.
  - Identification around a desired operation point, a controller is required to keep the system in the desired operation point.

Identification of a system in closed loop



- Closed loop: $S_{un}(f) \neq 0$!
- Consequently: $H(f) \neq S_{uy}(f)/S_{uu}(f)$

Identification in the closed loop

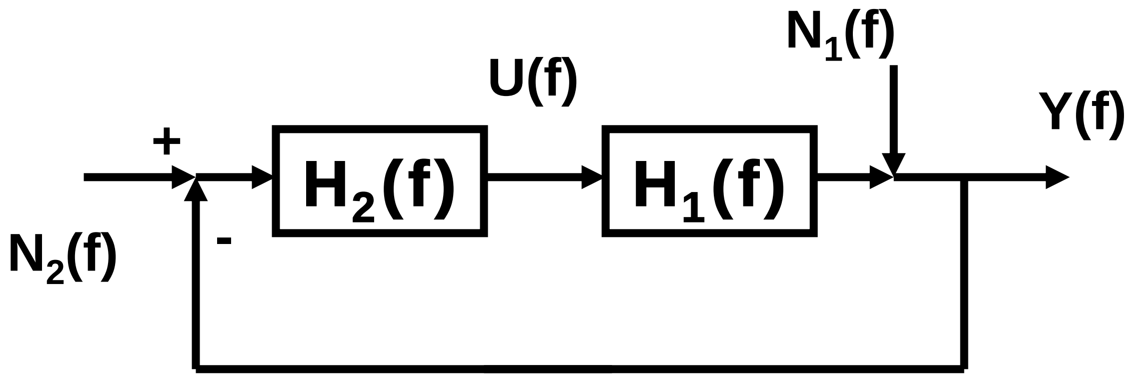


- What happens if we would use an open loop estimator for a system in closed loop:

$$H_1'(f) = \frac{S_{uy}(f)}{S_{uu}(f)}$$

- What is relation between H_1` and true H_1

Identification in the closed loop



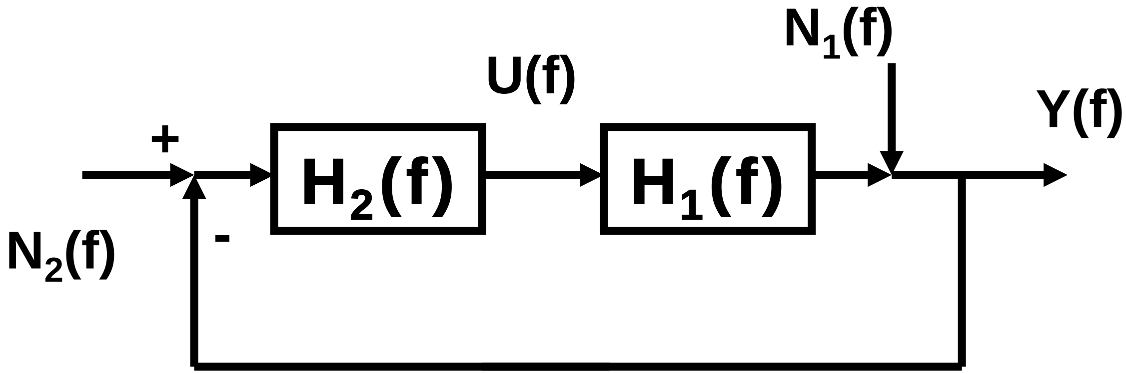
- Write every signal (U,Y) as combination from the independent inputs: N_1 and N_2

$$H_1'(f) = -\frac{1}{H_2(f)} \frac{S_{N_1N_1}(f)}{S_{N_1N_1}(f) + S_{N_2N_2}(f)} + H_1(f) \frac{S_{N_2N_2}(f)}{S_{N_1N_1}(f) + S_{N_2N_2}(f)}$$

- If $S_{n2n2} >> S_{n1n1}$: $H_1' = H_1$

- If $S_{n1n1} >> S_{n2n2}$: $H_1' = -1/H_2$ => inverse of feedback path!

Derivation



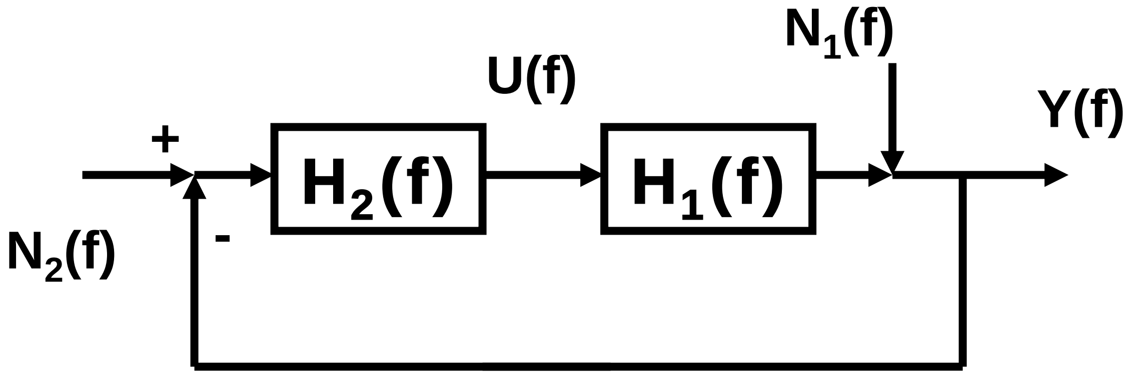
$$Y(f)=\frac{1}{1+H_1(f)H_2(f)}N_1(f)+\frac{H_1(f)H_2(f)}{1+H_1(f)H_2(f)}N_2(f)$$

$$U(f)=\frac{-H_2(f)}{1+H_1(f)H_2(f)}N_1(f)+\frac{H_2(f)}{1+H_1(f)H_2(f)}N_2(f)$$

$$S_{uy}(f)=\left|\frac{1}{1+H_1(f)H_2(f)}\right|^2\left\{-H_2(-f)S_{N_1N_1}(f)+H_1(f)\left|H_2(f)\right|^2 S_{N_2N_2}(f)\right\}$$

$$S_{uu}(f)=\left|\frac{1}{1+H_1(f)H_2(f)}\right|^2\left\{\left|H_2(f)\right|^2 S_{N_1N_1}(f)+\left|H_2(f)\right|^2 S_{N_2N_2}(f)\right\}$$

Derivation



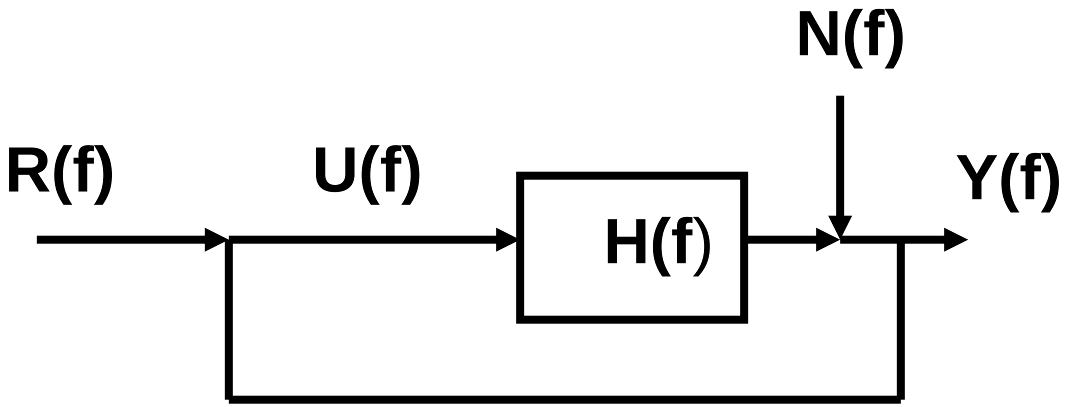
$$S_{uy}(f) = \left|\frac{1}{1+H_1(f)H_2(f)}\right|^2 \left\{-H_2(-f)S_{N_1N_1}(f) + H_1(f)\left|H_2(f)\right|^2 S_{N_2N_2}(f)\right\}$$

$$S_{uu}(f) = \left|\frac{1}{1+H_1(f)H_2(f)}\right|^2 \left\{\left|H_2(f)\right|^2 S_{N_1N_1}(f) + \left|H_2(f)\right|^2 S_{N_2N_2}(f)\right\}$$

$$H_1'(f) = \frac{S_{uy}(f)}{S_{uu}(f)} = \frac{-H_2(-f)S_{N_1N_1}(f) + H_1(f)\left|H_2(f)\right|^2 S_{N_2N_2}(f)}{\left|H_2(f)\right|^2 S_{N_1N_1}(f) + \left|H_2(f)\right|^2 S_{N_2N_2}(f)}$$

$$= -\frac{1}{H_2(f)} \frac{S_{N_1N_1}(f)}{S_{N_1N_1}(f) + S_{N_2N_2}(f)} + H_1(f) \frac{S_{N_2N_2}(f)}{S_{N_1N_1}(f) + S_{N_2N_2}(f)}$$

Identification of a system in closed loop



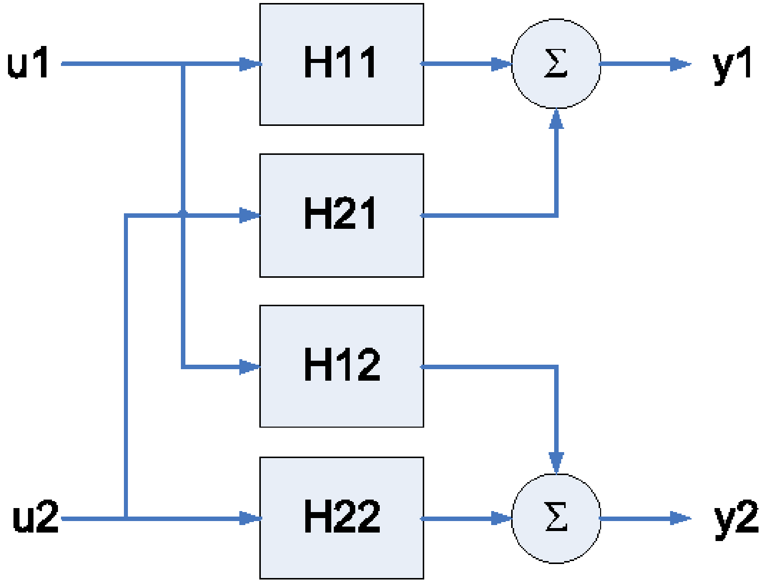
- Closed loop: $S_{un}(f) \neq 0$!
- Solution: use external input R(f): $S_{rn}(f) = 0$

$$\hat{H}_{uy}(f) = \frac{\hat{S}_{ry}(f)}{\hat{S}_{ru}(f)}$$

$$\hat{\gamma}_{ry}{}^2(f) = \frac{\left|\hat{S}_{ry}(f)\right|^2}{\hat{S}_{rr}(f)\hat{S}_{yy}(f)} \quad \left(or\ \hat{\gamma}_{ru}{}^2\right)$$

Multivariable system

- Most simple multivariable case: two inputs and two outputs

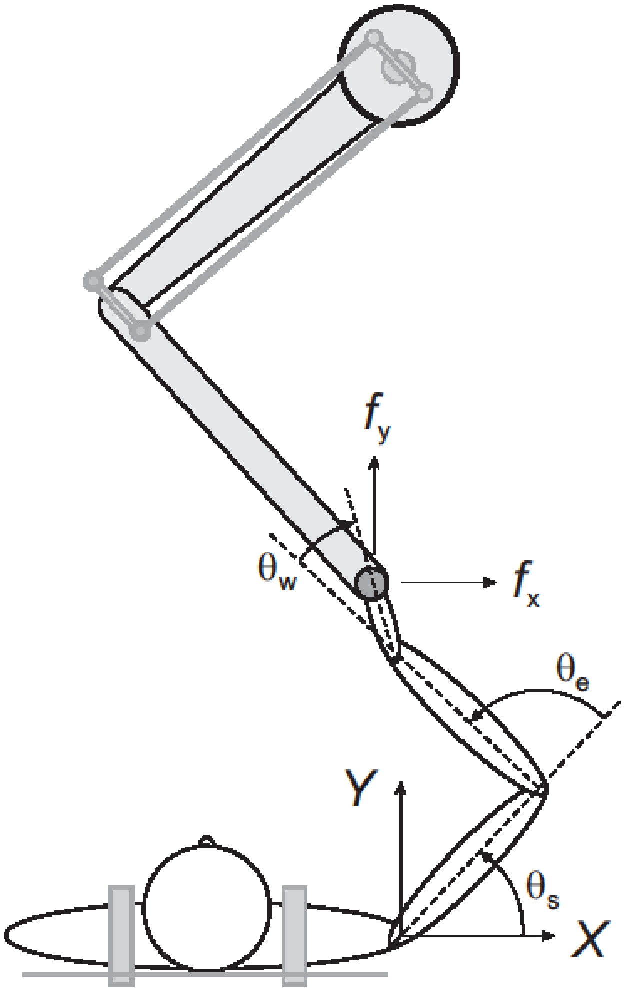


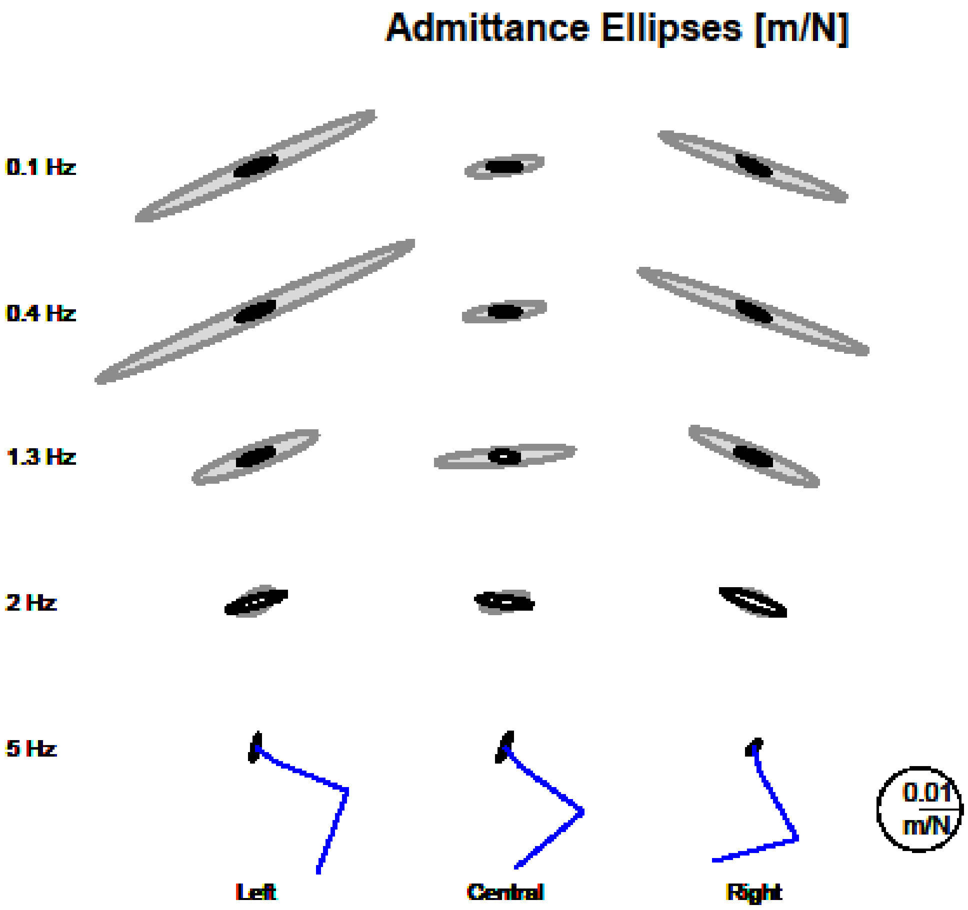
$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} H_{11} & H_{21} \\ H_{12} & H_{22} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$Y = HU$$

Note matrix element 12 represents the FRF from input 2 to output 1 and therefore denotes H_{21}

Example 2-DOF Arm Impedance





Estimator for multivariable FRF

$$Y = HU + N$$

$$S_{zy} = HS_{zu} + S_{zn}$$

- N: output noise
- Z: external (known) signal (instrument variable) such that $S_{zn}=0$

$$\hat{H} = \hat{S}_{zy}\hat{S}_{zu}^{-1}$$

- Open-loop: take Z=U

Estimator for multivariable FRF

$$\hat{H} = \hat{S}_{zy} \hat{S}_{zu}^{-1}$$

- For 2 input - 2 output system:

$$S_{zy} = \begin{bmatrix} S_{z1y1} & S_{z2y1} \\ S_{z1y2} & S_{z2y2} \end{bmatrix}$$

$$S_{zu} = \begin{bmatrix} S_{z1u1} & S_{z2u1} \\ S_{z1u2} & S_{z2u2} \end{bmatrix}$$

- H exists if S_{zu} is invertible (has full rank) for all frequencies

Rank input matrix S_{zu}

- If inputs are fully coupled: z1=L.z2
- Static or dynamic! Then S_{zu} is not invertible (loses rank)

$$S_{zu}^{-1} = \begin{bmatrix} S_{z1u1} & S_{z2u1} \\ S_{z1u2} & S_{z2u2} \end{bmatrix}^{-1}$$

$$= \frac{adj(S_{zu})}{\left|S_{zu}\right|}$$

$$= \begin{bmatrix} \dfrac{S_{z2u2}}{S_{z1u1}S_{z2u2} - S_{z1u2}S_{z2u1}} & \dfrac{-S_{z2u1}}{S_{z1u1}S_{z2u2} - S_{z1u2}S_{z2u1}} \\ \dfrac{-S_{z1u2}}{S_{z1u1}S_{z2u2} - S_{z1u2}S_{z2u1}} & \dfrac{S_{z1u1}}{S_{z1u1}S_{z2u2} - S_{z1u2}S_{z2u1}} \end{bmatrix}$$

Estimator for multivariable FRF

$$\hat{H} = \hat{S}_{zy}\hat{S}_{zu}^{-1}$$

$$= \begin{bmatrix} S_{z1y1} & S_{z2y1} \\ S_{z1y2} & S_{z2y2} \end{bmatrix} \begin{bmatrix} \dfrac{S_{z2u2}}{S_{z1u1}S_{z2u2} - S_{z1u2}S_{z2u1}} & \dfrac{-S_{z2u1}}{S_{z1u1}S_{z2u2} - S_{z1u2}S_{z2u1}} \\ \dfrac{-S_{z1u2}}{S_{z1u1}S_{z2u2} - S_{z1u2}S_{z2u1}} & \dfrac{S_{z1u1}}{S_{z1u1}S_{z2u2} - S_{z1u2}S_{z2u1}} \end{bmatrix}$$

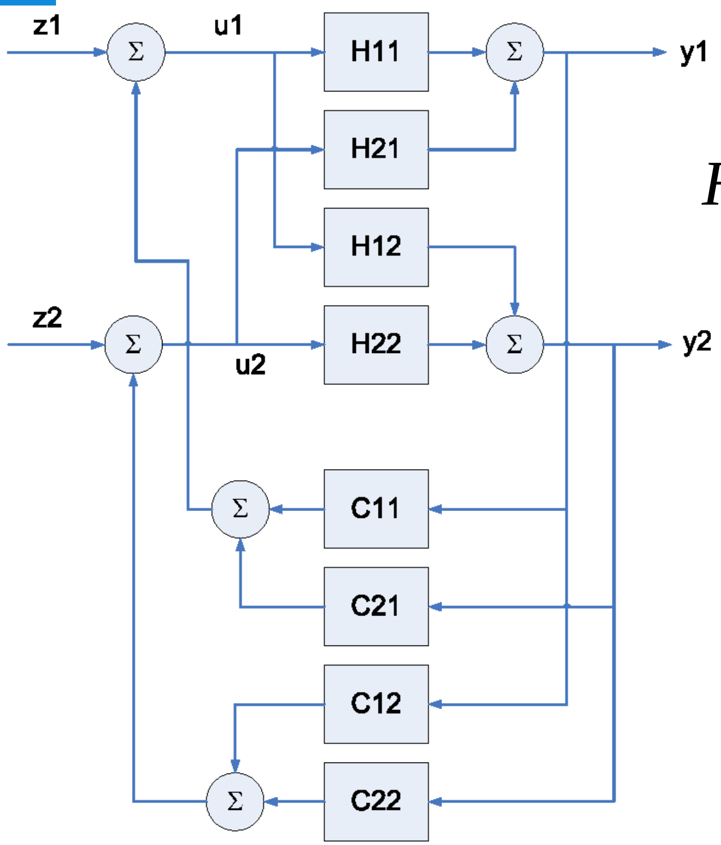
$$= \begin{bmatrix} \dfrac{S_{z1y1}S_{z2u2} - S_{z2y1}S_{z1u2}}{S_{z1u1}S_{z2u2} - S_{z1u2}S_{z2u1}} & \dfrac{-S_{z1y1}S_{z2u1} + S_{z2y1}S_{z1u1}}{S_{z1u1}S_{z2u2} - S_{z1u2}S_{z2u1}} \\ \dfrac{S_{z1y2}S_{z2u2} - S_{z2y2}S_{z1u2}}{S_{z1u1}S_{z2u2} - S_{z1u2}S_{z2u1}} & \dfrac{-S_{z1y2}S_{z2u1} + S_{z2y2}S_{z1u1}}{S_{z1u1}S_{z2u2} - S_{z1u2}S_{z2u1}} \end{bmatrix}$$

Estimator for multivariable FRF

$$\hat{H} = \begin{bmatrix} \dfrac{S_{z1y1}S_{z2u2} - S_{z2y1}S_{z1u2}}{S_{z1u1}S_{z2u2} - S_{z1u2}S_{z2u1}} & \dfrac{-S_{z1y1}S_{z2u1} + S_{z2y1}S_{z1u1}}{S_{z1u1}S_{z2u2} - S_{z1u2}S_{z2u1}} \\ \dfrac{S_{z1y2}S_{z2u2} - S_{z2y2}S_{z1u2}}{S_{z1u1}S_{z2u2} - S_{z1u2}S_{z2u1}} & \dfrac{-S_{z1y2}S_{z2u1} + S_{z2y2}S_{z1u1}}{S_{z1u1}S_{z2u2} - S_{z1u2}S_{z2u1}} \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{S_{z1y1} - S_{z2y1}\dfrac{S_{z1u2}}{S_{z2u2}}}{S_{z1u1} - S_{z2u1}\dfrac{S_{z1u2}}{S_{z2u2}}} & \dfrac{S_{z2y1} - S_{z1y1}\dfrac{S_{z2u1}}{S_{z1u1}}}{S_{z2u2} - S_{z1u2}\dfrac{S_{z2u1}}{S_{z1u1}}} \\ \dfrac{S_{z1y2} - S_{z2y2}\dfrac{S_{z1u2}}{S_{z2u2}}}{S_{z1u1} - S_{z2u1}\dfrac{S_{z1u2}}{S_{z2u2}}} & \dfrac{S_{z2y2} - S_{z1y2}\dfrac{S_{z2u1}}{S_{z1u1}}}{S_{z2u2} - S_{z1u2}\dfrac{S_{z2u1}}{S_{z1u1}}} \end{bmatrix}$$

Closed loop case



$$\hat{H} = \begin{bmatrix} \dfrac{S_{z1y1} - S_{z2y1}\dfrac{S_{z1u2}}{S_{z2u2}}}{S_{z1u1} - S_{z2u1}\dfrac{S_{z1u2}}{S_{z2u2}}} & \dfrac{S_{z2y1} - S_{z1y1}\dfrac{S_{z2u1}}{S_{z1u1}}}{S_{z2u2} - S_{z1u2}\dfrac{S_{z2u1}}{S_{z1u1}}} \\ \dfrac{S_{z1y2} - S_{z2y2}\dfrac{S_{z1u2}}{S_{z2u2}}}{S_{z1u1} - S_{z2u1}\dfrac{S_{z1u2}}{S_{z2u2}}} & \dfrac{S_{z2y2} - S_{z1y2}\dfrac{S_{z2u1}}{S_{z1u1}}}{S_{z2u2} - S_{z1u2}\dfrac{S_{z2u1}}{S_{z1u1}}} \end{bmatrix}$$

Open loop: particular case

$$Z = U: \qquad \hat{H} = \begin{bmatrix} \dfrac{S_{u1y1} - S_{u2y1}\dfrac{S_{u1u2}}{S_{u2u2}}}{S_{u1u1} - S_{u2u1}\dfrac{S_{u1u2}}{S_{u2u2}}} & \dfrac{S_{u2y1} - S_{u1y1}\dfrac{S_{u2u1}}{S_{u1u1}}}{S_{u2u2} - S_{u1u2}\dfrac{S_{u2u1}}{S_{u1u1}}} \\ \dfrac{S_{u1y2} - S_{u2y2}\dfrac{S_{u1u2}}{S_{u2u2}}}{S_{u1u1} - S_{u2u1}\dfrac{S_{u1u2}}{S_{u2u2}}} & \dfrac{S_{u2y2} - S_{u1y2}\dfrac{S_{u2u1}}{S_{u1u1}}}{S_{u2u2} - S_{u1u2}\dfrac{S_{u2u1}}{S_{u1u1}}} \end{bmatrix}$$

- If inputs are decoupled, i.e. $S_{u1u2}=0$, then:

$$\hat{H} = \begin{bmatrix} \dfrac{S_{u1y1}}{S_{u1u1}} & \dfrac{S_{u2y1}}{S_{u2u2}} \\ \dfrac{S_{u1y2}}{S_{u1u1}} & \dfrac{S_{u2y2}}{S_{u2u2}} \end{bmatrix}$$

Coherence functions

- Ordinary coherence: from one input to one output
- Multiple coherence: from all input to one output
- Partial coherence: from one input to one output, while all contributions from other inputs are removed from both input and output of interest

Ordinary coherence

- The H_{11} case

$$\hat{\gamma}_{z1y1}{}^{2} = \frac{\left|S_{z1y1}\right|^{2}}{S_{z1z1}S_{y1y1}}$$

- Always smaller than one (if power of other inputs > 0), even when there is no noise!
- No compensation for input coupling

Multiple Coherence

- From all input to one output, case for the first output:

$$\hat{\gamma}_{zy1}^{\;2} = \frac{conj\left(H_{z1y1}\right)S_{z1y1} + conj\left(H_{z2y1}\right)S_{z2y1}}{S_{y1y1}}$$

$$= \frac{H_{11}^{\;*}S_{z1y1} + H_{21}^{\;*}S_{z2y1}}{S_{y1y1}}$$

- Includes estimations of the individual transfer functions, so there is compensation for input coupling
- Unknown from which subsystem the noise originates

Partial Coherence

- For the H_{11} case:

$$\hat{\gamma}_{z1y1\bullet z2}{}^2 = \frac{\left|S_{z1y1\bullet z2}\right|^2}{S_{z1z1\bullet z2}S_{y1y1\bullet z2}}$$

- The bullet denotes removal of all other inputs (z2) within the estimates

Partial Coherence

$$\hat{\gamma}_{z1y1\bullet z2}^{2} = \frac{\left|S_{z1y1\bullet z2}\right|^{2}}{S_{z1z1\bullet z2}S_{y1y1\bullet z2}}$$

$$= \frac{\left|S_{z1y1}S_{z2z2} - S_{z2y1}S_{z1z2}\right|^{2}}{\left(S_{z2z2}\right)^{2} S_{z1z1}S_{y1y1}\left(1-\gamma_{z2z1}^{2}\right)\left(1-\gamma_{z2y1}^{2}\right)}$$

Overview course: lectures 1-5

- Non-parametric system identification (lecture 1-5)
  - Correlation functions: $C_{uu}(\tau)$, $C_{uy}(\tau)$
  - Impulse response function: $h(t)$
  - Spectral density functions: $S_{uu}(f)$, $S_{uy}(f)$, $S_{yy}(f)$
  - Frequency response function, coherence: $H(f)$ $\gamma^2(f)$
  - Open-loop vs closed-loop
  - Multivariable

- Input-output model: ARX, ARMAX, OE, BJ (lecture 6-7)
  - ARX, ARMAX, OE, BJ
  - Open-loop and closed-loop

- Optimization methods, residue analysis (lecture 8-9)
- Non linear, non-parametric system identification (lecture 11-12)
- Case, overview, exam (lecture 14)

Readings

- Book Westwick & Kearney
  - Chapter 1, all (lecture 1)
  - Chapter 2, sec. 2.1 – 2.3.4 (lecture 1+2)
  - Chapter 3, sec. 3.1 – 3.2 (lecture 2)
  - Chapter 5, sec. 5.1 – 5.3 (lecture 3)

- Book Pintelon & Schoukens
  - Chapter 1, sec 1.1 – 1.4 (optional, lecture 1)
  - Chapter 2, all (lecture 4)
  - Chapter 4, all (lecture 4)

- Articles
  - de Vlugt et al. (lecture 5)