System Identification & Parameter Estimation

Wb2301: SIPE

Lecture 10: Nonlinear Models

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May 10 2010

Contents

- Nonlinear behavior
- Harmonics
- The Volterra Series
- L-N structured models
    - Book: refer to parts from Chapter 4 of Westwick and Kearney

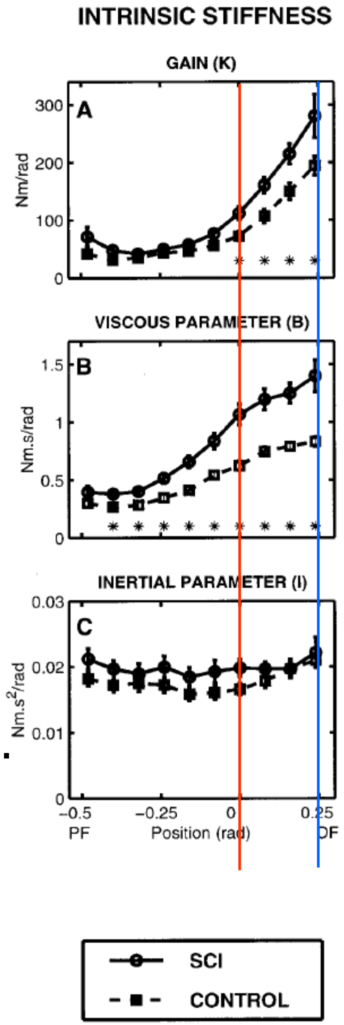
Nonlinear behavior

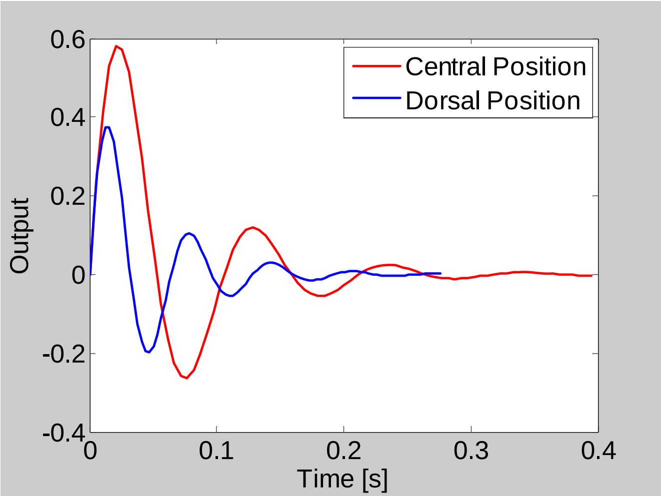
- Static nonlinearity
  - ‘memory-less’ relationship between variables
  - continuous, e.g. $y = u^2$, $y = e^u$
  - discontinuous, e.g. y = y for y >= 0, otherwise y = 0
  - e.g. stress-strain relationship (stiffness) of (bio-)materials
  - polynomial description of variable

- Dynamic nonlinearity
  - e.g. $y(t) = u^2(t) / (s + 1)$
  - continuous and discontinuous
  - e.g. stiffness as part of the larger neuro-mechanical joint system
  - polynomial description of dynamics

Examples Nonlinear properties

- Passive ankle stiffness
  - exponential stress-strain of muscular tissues
- Passive ankle viscosity
  - exponential increase in viscosity, likely from fluid displacement, stiction etc.





from Mirbagheri 2001

Harmonics

- static nonlinearity can be described in the frequency domain using harmonics

- e.g. square function $y = u^2$
- $u = \sin(wt)$ (fundamental frequency, 1st harmonic)
- $y = \sin^2(wt) = 0.5 - 0.5\cos(2wt)$
- double frequency (2nd harmonic) in output (plus constant)

- each ‘theoretical’ static nonlinearity has its own (‘fingerprint’) harmonics

Linear Convolution

$$y(t) = \int_0^t h(\tau)u(t-\tau)d\tau$$

- Additivity (superposition) holds:

$$y_1(t) = N(u_1(t))$$
$$y_2(t) = N(u_2(t))$$
$$y_1(t) + y_2(t) = N(u_1(t) + u_2(t))$$

- Homogeneity holds:

$$y(t) = N(\alpha u(t)) = \alpha N(u(t))$$

Nonlinear convolution equation

- Example, 2nd order nonlinear system:

$$y(t) = N(u(t)) = \int_0^T \int_0^T h^{(2)}(\tau_1, \tau_2) u(t-\tau_1) u(t-\tau_2) d\tau_1 d\tau_2$$

- What kind of system is this? Impulse response:

$$y(t) = N(\delta(t)) = h^{(2)}(t,t)$$

- Additivity and Homogeneity do not hold, e.g.:

$$y(t) = N(\alpha u(t)) = \alpha^2 N(u(t))$$

Volterra series

$$y(t)=\sum_{q=0}^{\infty}\int_0^{\infty}\ldots\int_0^{\infty}h^{(q)}\left(\tau_1,\ldots,\tau_q\right)u(t-\tau_1)\ldots u(t-\tau_q)d\tau_1\ldots d\tau_q$$

- *q* is the order of the nonlinearity
- *q* = 0, constant term independent of input

$$y^{(0)}(t)=h^{(0)}$$

- *q = 1*

$$y^{(1)}(t)=\int_0^t h^{(1)}(\tau)u(t-\tau)d\tau$$

- Impulse response of first order (linear) system (if all higher order kernels are zero):

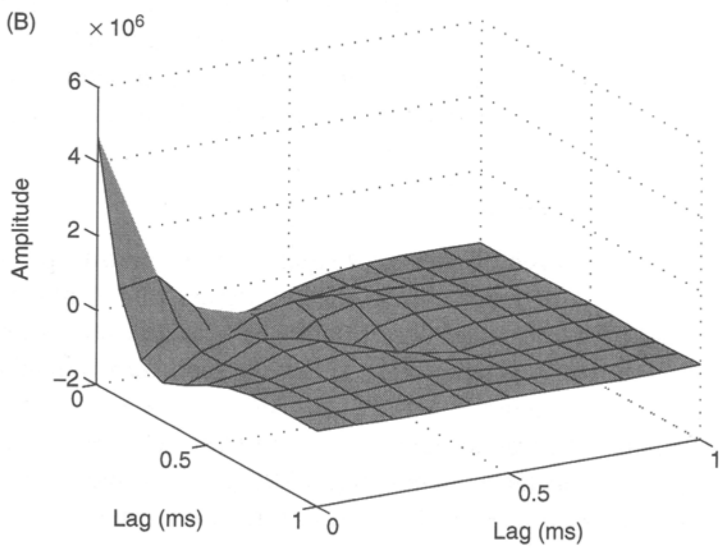
$$y^{(0-1)}(t)=h^{(0)}+h^{(1)}(t)$$

General system impulse response

- Finite Volterra series: Q kernels

$$y(t) = h^{(0)} + h^{(1)}(t) + h^{(2)}(t) + \ldots + h^{(Q)}(t)$$

Kernel Symmetry



- second order (q = 2) and higher kernels are diagonal symmetric:

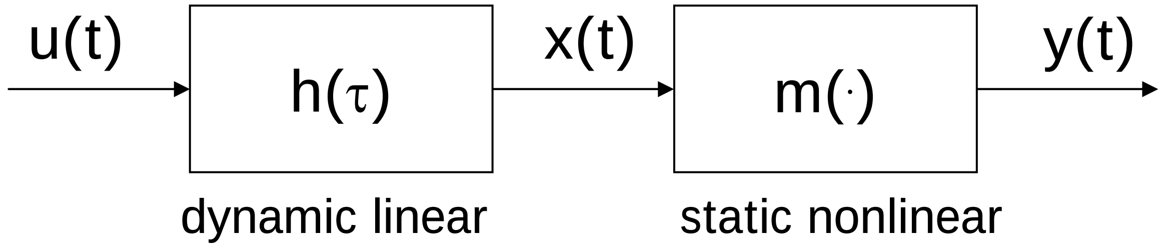
$$h^{(2)}(\tau_1, \tau_2) = h^{(2)}(\tau_2, \tau_1)$$

- since interchanging indices is equal to interchanging the two copies inputs u(t - τ_1) and u(t - τ_2)
- diagonal describes components that are the power q of the input

Block structures

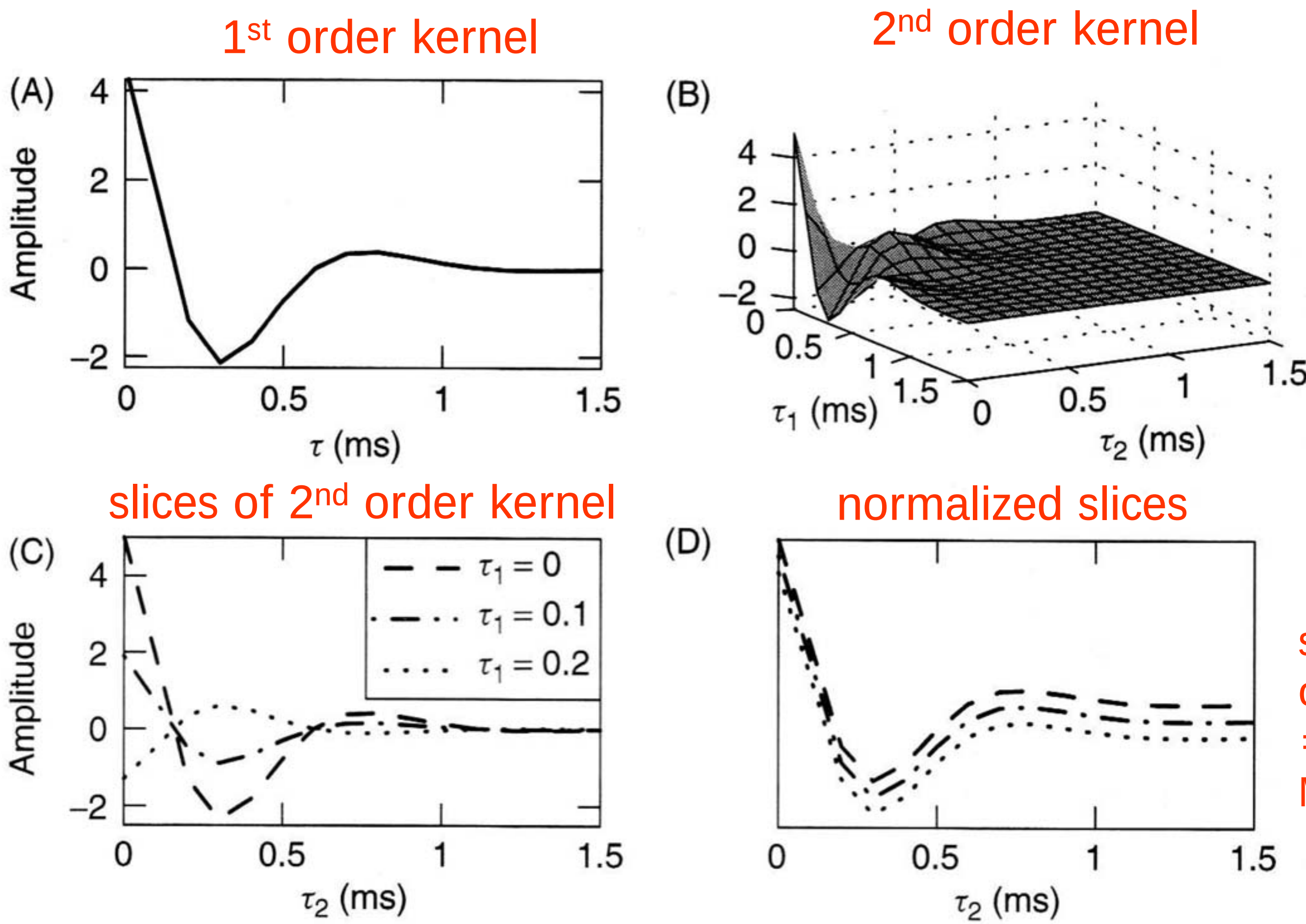
- Volterra series represent a wide variety of systems
- However, expressions are cumbersome
- Use simple models consisting of linear and static nonlinearities **in series** => efficient descriptions of limited class of nonlinear systems, e.g.:
  - Wiener: Linear (L) – Static Nonlinear (N)
  - Hammerstein: N-L
  - Other combinations

Wiener Model



- nonlinearity: $m(x(t)) = \sum_{q=0}^{Q} c^{(q)} x^{q}(t)$
- output: $y(t) = \sum_{q=0}^{Q} c^{(q)} \left(\sum_{\tau=0}^{T-1} h(\tau) u(t-\tau) \right)^{q}$
- relation to Volterra kernels: $h^{(q)}(\tau_1, \ldots, \tau_q) = c^{(q)} h(\tau_1) h(\tau_2) \ldots h(\tau_q)$
  - 1-dimensional slice (parallel to axis of Volterra kernel) is proportional to h(τ)!, e.g. $h^{(2)}(\tau_1, k) = c^{(2)} h(\tau_1) h(k)$

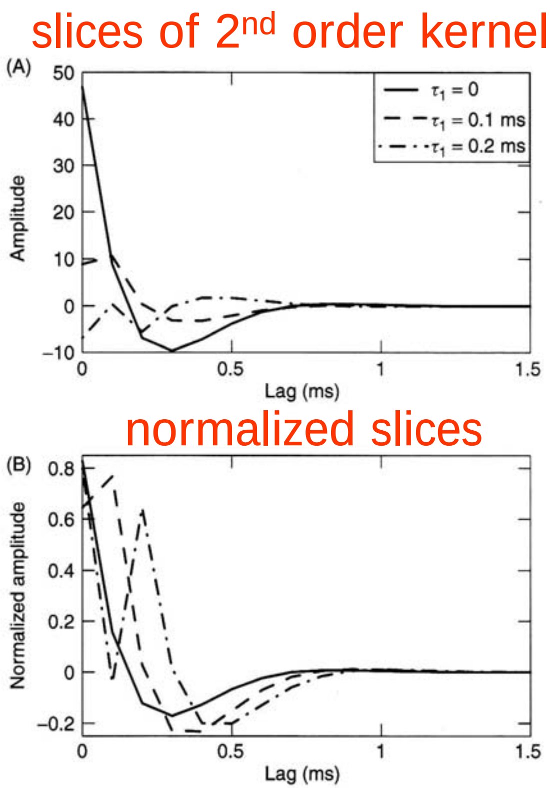
Testing for Wiener Structure



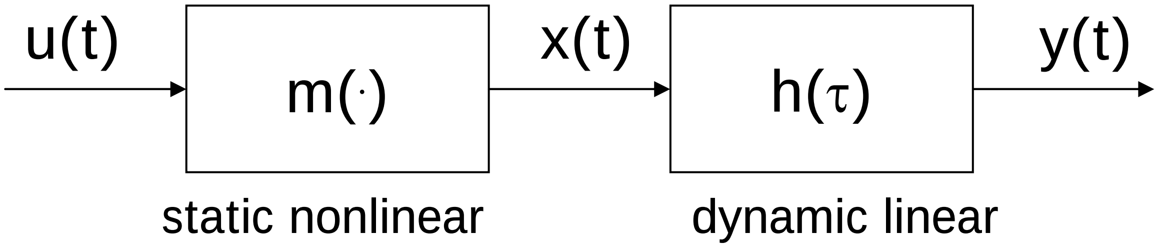
scaled versions of each other => Wiener Model

Testing for Wiener Structure

slices of 2nd order kernels are not scaled versions of each other
=> this system can not be described properly by a Wiener model



Hammerstein Model



- output: $$y(t) = \sum_{\tau=0}^{T-1} h(\tau) \left\{ \sum_{q=0}^{Q} c^{(q)} u^q (t-\tau) \right\}$$
- expanding input power series:

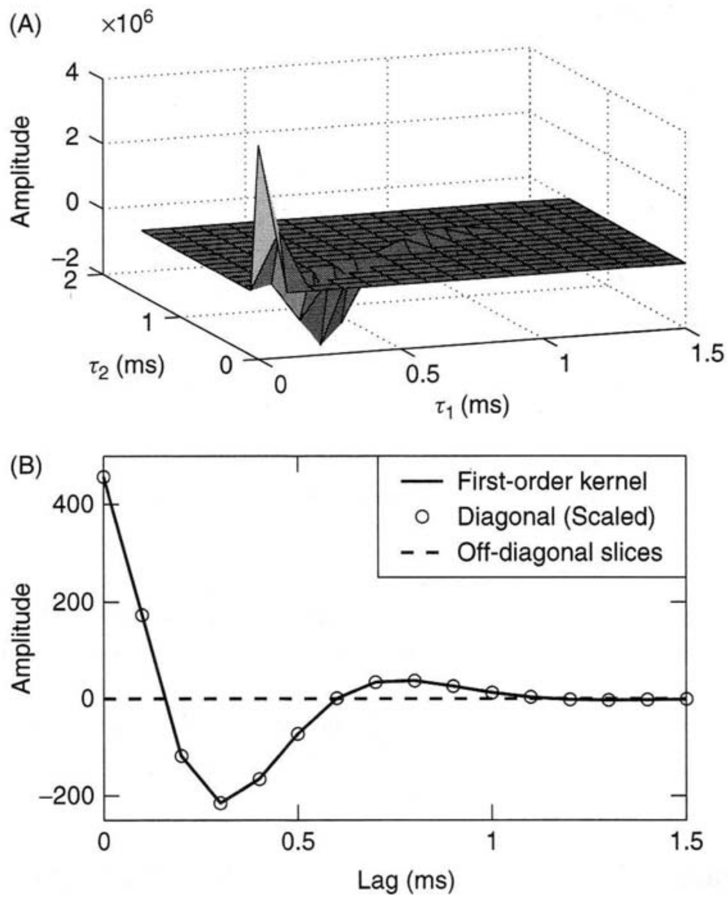
$$u^q(t-\tau) = u(t-\tau_1)u(t-\tau_2)\ldots u(t-\tau_q)\delta_{\tau_1\tau_2\ldots\tau_q}$$

with $\delta_{\tau_1\tau_2\ldots\tau_q}$ the Kronecker delta (multidimensional)

- Volterra kernels:
  - nonzero only at diagonal where $\tau_1 = \tau_2 = \ldots = \tau_q$

Test for Hammerstein Structure

- System with Hammerstein structure

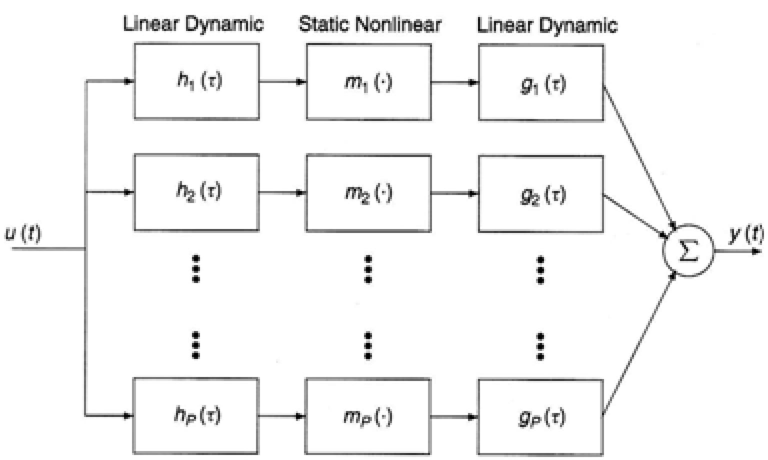


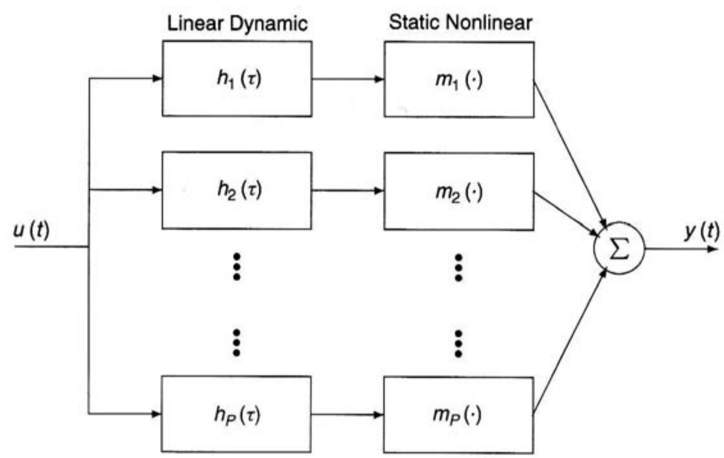
Other model combinations

- L-N-L Wiener-Hammerstein
- N-L-N
- see par. 4.3.3 and 4.4.4 in Westwick and Kearney

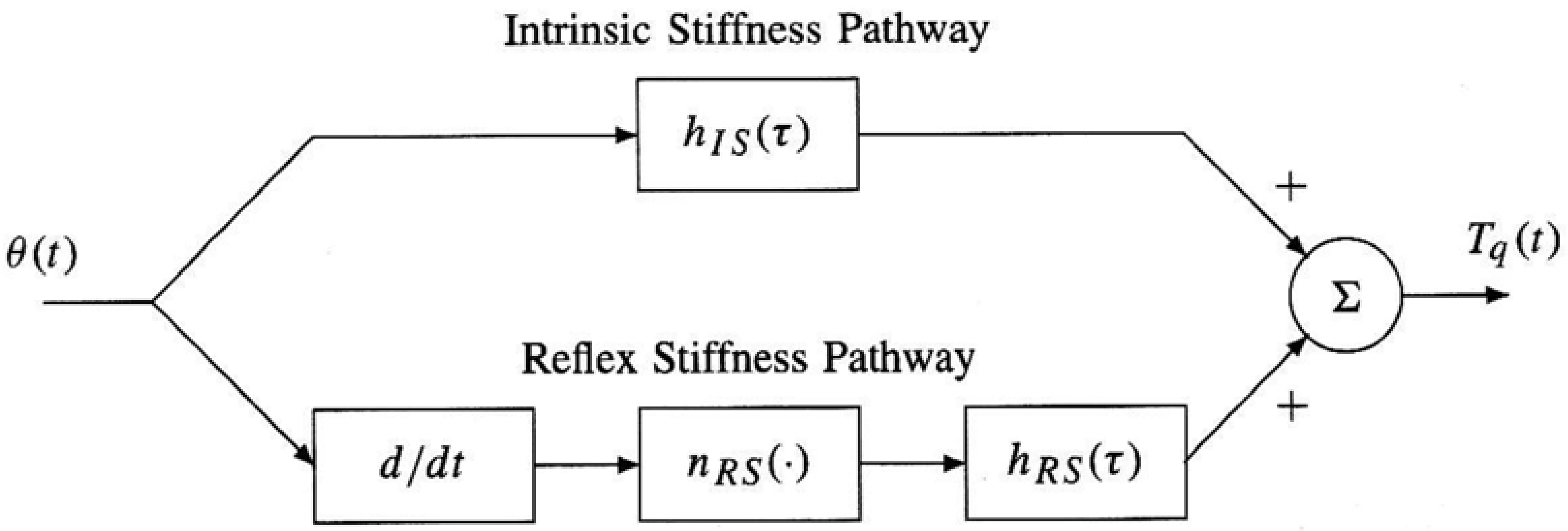
Parallel Cascades

- Wiener and Hammerstein models are serial cascades
- Many system require parallel structures





Example Structure: Impedance of the human ankle joint



Summary

Current models can be grouped into three basic classes

- Parametric approaches (e.g. Physical ODEs)
- Cascade or block structured techniques (e.g. Hammerstein, Wiener, LNL structures)
- Nonparametric kernel or functional series approaches (e.g. Wiener and Volterra representations)

1. The parametric methods have the advantage of producing very accurate descriptions of system behavior but require considerable *a priori* knowledge about system structure and order.
2. The cascade and kernel approaches are less efficient but are attractive for the investigation of unknown systems because their success is not dependent upon *a priori* information.