3 Mathematical Descriptions of Processes: Models

This chapter presents the reader with the basic definition of a model and the different classes of models that can be developed. The material in this chapter will enable the user to understand the various factors that characterize a model and the criteria that govern its choice.

A fact that stands out distinctly in Chapters 1 and 2 is that the *model* is both the end-product and the centerpiece of every identification exercise. A good model necessarily requires "good" quality data and the choice of a good starting model. While generating quality data is achieved by a proper input design, the choice of the initial model structure depends on the a priori knowledge of the suitable model type and structure. Usually the situation is that we begin with a good *guess* and improve upon the initial choice if necessary (as in the case study of Chapter 2). The final decision is governed by how well the model suits the purpose, cost (of estimation) considerations, the merits and limitations of a given model and other constraints.

The specification of a model is usually in terms of its *type*, *structure* and *order*. Any blind guess of these specifications will obviously take us through a circuitous path and at times not even converge to a working model. A good guess should be based on any a priori knowledge of the process complexity, a preliminary examination of the data, ease of estimation, end-use and physical meaningfulness of the model. A systematic approach can be expected to converge to a useful working model in a minimum number of steps.

The first step in this process is to obtain an overview of the different model classes available to the user. This chapter is devoted to this cause.

We begin with the formal definition of a model.

3.1 DEFINITION OF A MODEL

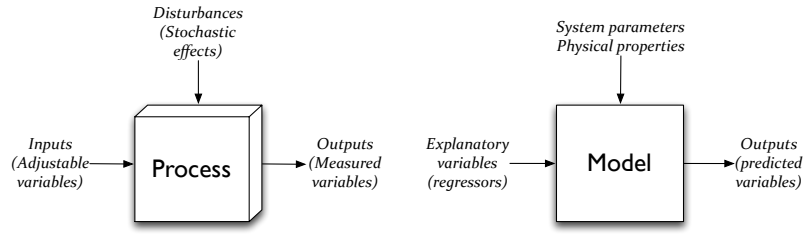
A model is a set of linear/non-linear (ordinary or partial) differential (or difference), algebraic equations in terms of the process variables / states, inputs, and parameters, built on a set of *assumptions*.

The model is essentially a mathematical abstraction of the physical process. Later in this text, in Chapter 18, we shall observe that an alternative way of specifying a model is to specify the predictor (prediction expression) and characterize the prediction error.

Terminology

In modeling literature, *inputs* to a model need not correspond to the physical inputs to a system. Similarly, *outputs* need not correspond to the physical outputs of the system. They have a broader connotation. Outputs of a model constitute those variables that we wish to *predict* or *explain*, while inputs to a model include all those variables that participate in the prediction. For this reason, model inputs are more appropriately known as *explanatory variables*.

Figure 3.1 schematically shows the difference in the workings of a process and a model. It highlights the fact that the inputs to the model are not merely the physical inputs to the process. The



(a) Process generates outputs (b) Model predicts outputs

FIGURE 3.1 The inputs and outputs of a model are in general different from the physical inputs and outputs of a process.

model input is a broader set. It consists of the past and present inputs to the process and all other variables that participate in the prediction of the outputs of interest. An important part of any model that should never be forgotten is the set of underlying assumptions. A popular class of descriptions known as state-space models use an additional set of variables known as *states* (see Chapter 4) to describe the process. These states are usually internal dynamical variables of a system. Section 4.4.3.1 discusses the related concepts in great detail.

A model is thus a map between the set of explanatory variables and the set of predicted variables.

Example 3.1: Explanatory Variables

Auto-regressive time-series models (see Section 9.5) have the form

$$v[k] = -d_1 v[k-1] - d_2 v[k-2] - \cdots - d_p v[k-p] + e[k] \qquad d_i \in \mathbb{R} \tag{3.1}$$

where $v[k]$ and $e[k]$ are the output and stochastic (endogenous) input of the random process, respectively. The explanatory variables in these models are the past outputs whereas in moving average models (see Section 9.4), the inputs are endogenous white-noise (shock-wave like) signals. On the other hand, where the model is used as a soft-sensor the inputs are all those variables that can aid in inference of the infrequently measured variables.

Notation: The text employs the conventional notation. Outputs and inputs are denoted by y and u, respectively, while the states are denoted by x.

3.2 CLASSIFICATION OF MODELS

The primary classification of models is based on the approach to modeling itself, i.e., **first-principles** models vs. **empirical** models. Within these two classes, further categorizations are possible based on several considerations, such as

- Do we wish to model the steady-state behavior or the transient (dynamic) behaviour?
- Are the important process variables dependent on process directions other than time? (e.g., is the concentration in a reactor varying with the location or can be assumed to be uniform?)
- Does the process of interest possess purely random behavior or does it exhibit a mix of deterministic and random behavior?
- What kind of input-output relationships are desirable? (e.g., linear, non-linear, time-varying, etc.)
- What is the end-use of the model? Do we wish to use the model for control, monitoring, optimization or prediction?

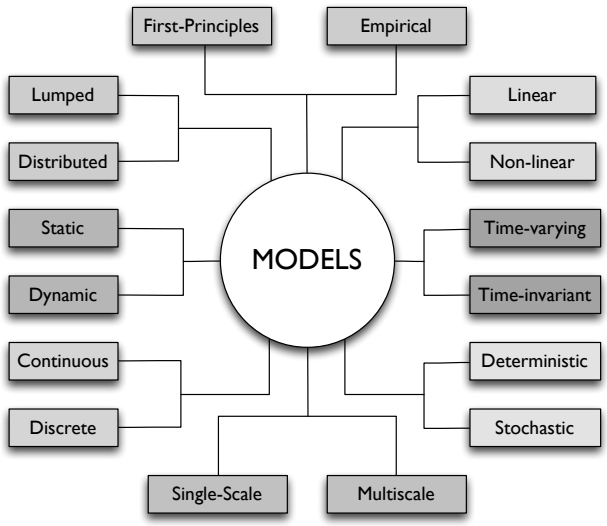


FIGURE 3.2 (**SEE COLOR INSERT**) Types of models.

- Should the model be physically meaningful or interpretable? Model structure may be required to follow a form known from prior knowledge, for example, from a first-principles description.

Following the line of thought above, models can be classified based on

i. **Approach** to modeling (e.g., first-principles vs. empirical)
ii. **System characteristics** (e.g., linear/non-linear, time-varying/time-invariant)
iii. **Knowledge** available to the user (e.g., deterministic/stochastic, black-box/grey-box)
iv. **Domain** of modeling (e.g., continuous-time/discrete-time, time-/frequency-domain)
v. **Response** characteristics (e.g., static/dynamic, lumped/distributed)

3.2.1 TYPES OF MODELS

The different types of models are shown in Figure 3.2. This classification is by no means hierarchical and exhaustive. We could begin with any class shown in Figure 3.2 and further sub-divide into remaining classes. For example, one can have a discrete-time, first-principles, non-linear model or a discrete-time, empirical, linear model. In addition to the types shown, we have *input-output* models and *state-space* models. However, this classification is based on the mathematical abstraction of the process.

We shall now briefly understand the nomenclature of the model classes shown in Figure 3.2.

In Section 1.1, a brief discussion concerning first-principles and empirical models was outlined. The section below offers more insights into this comparison while taking the opportunity to highlight a few important aspects.

First-principles vs. empirical models

This classification is based on the difference in the approach taken towards modeling. *First-principles* models are developed from fundamentals using basic laws and constitutive relationships. The approach generally results in *causal*, continuous, non-linear differential-algebraic equations. The equation in (2.6) is a simple example of a first-principles model. While these models are very effective and reliable, simulations of these models require good numerical ODE and algebraic solvers. The latter fact occludes first-principles models from being used in several on-line applications, for example, in control. Equally important is the fact that most processes do not offer the luxury of even

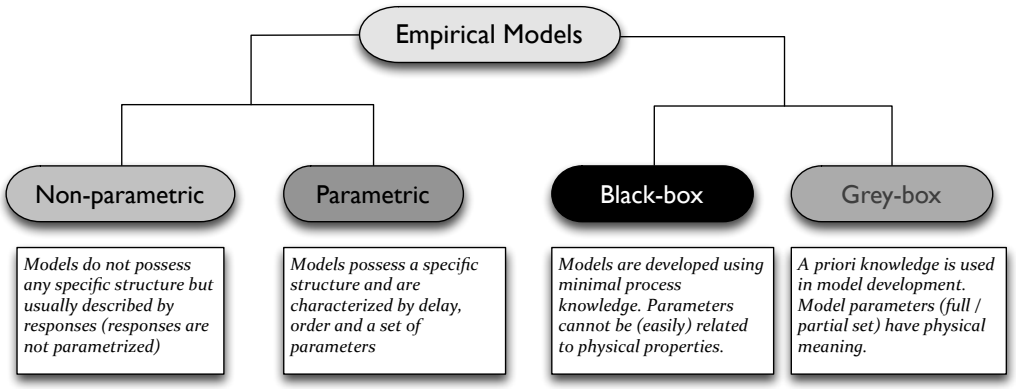


FIGURE 3.3 (**SEE COLOR INSERT**) Four broad categories of empirical models.

developing a first-principles model. The two foregoing facts are perhaps the most significant factors that have strongly impeded the popularity of this approach.

On the other hand, owing to their reliability, first-principles models are widely used in commercial simulators and in root-cause diagnosis. An important point to note is that *no model is truly first-principles*. All first-principles models possess some amount of empirical characteristics, with varying degrees, either in the form of simplifying assumptions or experimentally determined parameters.

Empirical models are built using measured data. Naturally, identified models are in discrete-time. Exceptions to this observation are where continuous-time models are directly identified from sampled data. One of the biggest benefits that empirical models hold over their first-principles counterparts is that only a minimal knowledge of process is required in developing them. The second advantage they offer is the flexibility in model structure, which is highly useful in several applications.

These benefits leverage on a few factors. Prime among them is the data quality, a factor that is not easy to control. While noise is effectively handled by the estimation methods available today, the main requirement is the presence of sufficient excitation in the inputs. Further, empirical models usually do not have good extrapolation capabilities. The model structure is chosen to best explain variations witnessed in the training data, which is usually a small subset of the entire range of possible process changes. The example of empirical modeling of the liquid level system in §2.4 may be recalled in this context. Another shortcoming of an empirical model is the inability to use it in understanding the effect of changing a process design parameter. For instance, the empirical model developed in §2.4 does not reveal how a change in cross-sectional area of the tank influences the liquid level. In contrast, first-principles models are naturally equipped to answer questions of this kind.

Notwithstanding the disadvantages, empirical models are very popular because of the ease with which they can be developed and the requirement of minimal process knowledge. For processes with complex behavior, empirical modeling is the natural alternative. Moreover, empirical models are usually built only in the post-design stage, i.e., these models are usually used only to capture process behavior for the chosen design conditions.

Classification of empirical models

The class of empirical models can be further subdivided into four broad categories, namely, parametric / non-parametric models (recall §1.3.3.2) and black-box / grey-box (recall §1.4.6). Figure 3.3 depicts these four categories along with their descriptions.

Non-parametric vs. parametric models

The distinction between a non-parametric and parametric model is based on whether the model possesses a specific "structure" chosen by the user or not.

Example 3.2: Non-Parametric Model: Impulse Response Model

Any discrete-time linear time-invariant system with input $u[k]$ and output $y[k]$ is described by the convolution equation

$$y[k] = \sum_{n=-\infty}^{\infty} g[n]u[k-n] \tag{3.2}$$

where $g[.]$ is known as the impulse response of the discrete-time system. Also known as the impulse response model, this is an example of a non-parametric model since the user makes no assumption about the "structure" of the model but rather only assumes linearity and time-invariance.

Chapters 4 and 17 discuss this model at length in the context of identification.

A common *misconception* is that non-parametric models are "free" of parameters or unknowns, which is obviously not true. The coefficients $\{g[k]\}$ are the parameters of the model, but bear a direct connection with the physical characteristics of the system.

The term parameter, in fact, stems from the *parametrization* of the system's response, typically to elementary inputs, for example, impulse, step and sinusoidal signals. As a consequence of this parametrization, the non-parametric response models can be re-expressed in terms of these parameters, resulting in what are known as parametric models. In the example below, the parametric model results when the impulse response sequence $\{g[k]\}$ of the convolution model (3.2) is parametrized (see Chapter 4 for details).

Example 3.3: Parametric Model: Difference Equation Model

Any discrete-time linear time-invariant system with input $u[k]$ and output $y[k]$ can be described by the difference equation

$$y[k] + \sum_{m=1}^{n_a} a_m y[k-m] = \sum_{n=0}^{n_b} b_n u[k-n] \tag{3.3}$$

where $\{a_m\}$ and $\{b_m\}$ are the parameters of the difference equation model.

In identification, the number of past inputs and outputs to be included become decision variables, which are usually determined by a mix of statistical tools and experience. Further, the parameters $\{a_m\}$ and $\{b_m\}$ do not directly reveal the characteristics of the system unlike $g[.]$, which directly reveals the impulse response of the system.

A formal definition of the term *structure* is presented in Chapters 17 and 18. For now it suffices to consider it as a term that refers to the specific mathematical form of the model and its parametrization. In the absence of any process knowledge, non-parametric models serve as good starting points in identification. The insight obtained from this exercise is used in the identification of a suitable parametric model. Chapters 4 and 17 contrast these two classes of models in greater detail.

Black-box vs. grey-box models

Traditionally empirical models have always been black-box models, meaning neither the structure nor the parameters have a one-to-one correspondence with the physical properties of the process. When the purpose of a model is only prediction, black-box models make a good choice. Where

physical interpretability is required, it becomes necessary to impose constraints on the structure (functional form) of the model and the nature of model, which is usually done before and/or during the estimation stage. The resulting models are then *grey-box* models, whose level of transparency depends on the a priori information available to the user.

For example, it is known that the displacement of a mass-spring-dampener system has second-order dynamics with respect to the external force applied. Imposing this requirement on the model order brings it closer to the underlying process and also allows a direct interpretation of the parameters of the model. Grey-box models, thus, offset certain disadvantages of an empirical model but at the expense of a more complicated and costly estimation exercise. Grey-box identification is an advanced and emerging topic in identification. In systems biology, grey-box models are often preferred to black-box models since scientists would like to use these mathematical models as means of understanding and making innovations to biological processes.

Linear vs. non-linear models

The classification is based on the mathematical nature of the assumption made about the system.

A system is said to be linear if and only if it satisfies the principles of homogeneity (scaling) and superposition (addition of inputs). A formal definition follows.

Definition 3.1. (Linear System) A system with input $u[k]$ and output $y[k]$

$$y[k] = \mathcal{T}\{u[k]\}$$

is linear if and only if

i. $\mathcal{T}\{\alpha u[k]\} = \alpha\mathcal{T}\{u[k]\} = \alpha y[k]$
ii. $\mathcal{T}\{\alpha_1 u_1[k] + \alpha_2 u_2[k]\} = \alpha_1 y_1[k] + \alpha_2 y_2[k]$

where $\mathcal{T}$ is the transformation operator.

Example 3.4: Linear System

$y = bu + a$ is linear only if $a = 0$; however, $(y - y_0) = b(u - u_0)$ is linear.

All systems that do not satisfy the linearity conditions are non-linear. No system in reality is exactly linear. However, for "small" changes in process conditions, the input-output relationship can be expected to exhibit linear behavior. The range over which this approximation holds good depends on the system in hand. Despite their obvious limitations, linear models are a natural choice in modeling; at least they serve as good starting points for a wide variety of processes. The linear approximation can be justified for several systems and applications where the approximation errors are efficiently handled by either a feedback strategy or an adaptive mechanism to account for changing operation conditions.

A major advantage of working with linear models is that the associated mathematics (of identification, control and estimation) is convenient and tractable. Moreover, they are transparent and easy to comprehend. Moreover, the theory for linear systems offers several insights into identification aspects that carry forward to non-linear systems. Another benefit is that several non-linear processes can be described by linear models built on transformed data.

As remarked in Chapter 1, this text is concerned with the development of linear models. Non-linear identification is a mathematically intricate area with relatively fewer applications. Section 25.2 describes a few popular methods used for identifying non-linear models.

Time-varying vs. time-invariant models

A system is said to be time-invariant if it produces the same output for the same input regardless of when the input was provided.

Mathematically,

$$\text{If } \; u[k] \xrightarrow{\mathcal{T}} y[k] \quad \text{then} \quad u[k-L] \xrightarrow{\mathcal{T}} y[k-L] \;\; \forall L \in Z$$

Practically, it means that the characteristics of the system do not change with time.

Example 3.5: Time-Invariant System

The difference equation model in (3.3) is an example of a time-invariant system. Observe that it is also linear.

Example 3.6: Time-Varying System

The system described by

$$y[k] + a_1[k]y[k-1] = b_0 u[k-1] \tag{3.4}$$

is an example of a time-varying system, where the time-varying nature is introduced by the time-dependence of the parameter a_1.

Observe that this is a linear time-varying system.

Remark: No system is truly time-invariant. However, several systems can be approximately thought of locally time-invariant, i.e., at the scale of observation / operation. Interestingly, the time-varying behavior can be sometimes lumped into random effects in the measurement or in the system.

As with the case of linear / non-linear models, time-invariant models are favorable starting points vis-a-vis time-varying models once again due to mathematical convenience and tractability. Similarly, several time-varying systems can be described as either time-invariant models in a transform domain or by a set of locally time-invariant models.

Deterministic vs. stochastic models

As we noted in §1.3.3.1, the measured response of any (physical) process can be split into two components:

1. A component associated with an external cause (designed or known). This is the *deterministic* component.
2. A component associated with disturbances, noise, unmeasured inputs, and modeling errors. This is the *stochastic* component.

A **deterministic model** predicts the response of a process *accurately*. These models are used to explain processes whose physics are accurately known. Furthermore, inputs to deterministic models are also deterministic, meaning their profiles are accurately known.

In contrast, a **stochastic model** is deployed to describe the response of a process which can *never be accurately* predicted. These models are built on the foundations of statistics and probability. The key challenge is that the observed response is one of the several possible responses due to which the optimal model is fit in a *statistical* sense as opposed to the optimal deterministic model which is fit in a *functional* sense. The resulting model is traditionally known as a *time-series model*. The theory of stochastic modeling is generally applied to forecast changes in any process for which no external cause could be associated or the cause itself is not measured or known.

The inputs to stochastic models, unlike in the case of deterministic models, are random signals, which assume values from a probability distribution. An encouraging aspect is that their statistical properties are fixed, which may have to be determined from data.

Example 3.7: Stochastic Model

A wide range of disturbances can be described by a first-order auto-regressive model

$$v[k] = -d_1 v[k-1] + e[k] \tag{3.5}$$

where $v[k]$ is the disturbance and $e[k]$ is the fictitious ideal random signal known as the white-noise signal (see Chapter 9).

The white-noise signal $e[k]$ is characterized by its statistical property known as the auto-correlation function (refer to Chapter 7).

Remark: No system is truly deterministic. An element of uncertainty always exists. However, from an engineering viewpoint systems are deterministic if the degree of predictability is very high. In identification, we usually build a composite model, i.e., a deterministic plus stochastic model. This discussion is continued in §3.2.2.

Single-scale vs. multiscale models

Several processes comprise phenomena that evolve over different time (or spatial) scales. A fuel cell system, atmospheric process, human system, and an integrated chemical process are all examples of such processes. In chemical engineering, the two time-constant (time-scale) process is a classical example of a multiscale system. Measurements of process variables contain contributions from sub-systems and (instrumentation) devices with significantly different time-constants. A fuel cell system exhibits multiscale behavior due to the large differences in the time-scales of the electrochemical sub-system (order of 10^{-5} s), the fuel flow sub-system (order of 10^{-1} s) and the thermal sub-system (order of 10^2 to 10^3 s). The atmospheric system is a complex, large, multiscale system consisting of micro-physical and chemical processes (order of 10^{-1}s) to temperature variations (order of hours) to seasonal variations (order of months). Modeling such systems presents huge challenges primarily because there is no single scale of observation that suits all the phenomena.

Models that are built over a single scale of observation (in time or space) are known as *single-scale* models. *Multiscale* models on the other hand model the system at different scales or layers. The subject of multiscale empirical modeling has gathered momentum only in the last decade. Majority of the literature focuses on single-scale models, which pose sufficient challenges in themselves. When not specified otherwise, any model is a single-scale model. Empirical models are built at a scale which is determined by the sampling interval. The text is limited to the development of single-scale empirical models only.

Continuous time vs. discrete-time models

As the names suggest, the classification is based on the nature of domain in which the models are developed. The inputs, outputs (and states) are all discrete-time quantities. Empirical models are usually developed in discrete-time since measurements are available in the form of sampled-data. Most of the applications are tailored to directly deploy these discrete-time models without having to estimate the continuous-time model of the process.

Nevertheless, there may arise situations where estimating the continuous-time models become necessary. To address these needs, various methods that can estimate the continuous-time models from discrete-time data have evolved. Continuous-time identification belongs to the category of modern identification areas, which is beyond the scope of this text.

Static vs. dynamic models

The categorization of models into *static* and *dynamic* models is a *response*-based one. Static models, also known as *steady-state* models relate instantaneous quantities, or at best describe the effects of delayed inputs.

Example 3.8: Static Model

The liquid level in a buffer system is related to the outlet flow rate through the valve equation at steady-state

$$F_{\text{out}}(t) = C_v \sqrt{h(t)} \tag{3.6}$$

The coefficient C_v is also known as the valve sizing coefficient.

Steady-state models do not have a "memory" associated with them.

Dynamic models are a more general class of models which describe the transient behavior of a process. These models are required in a wide range of applications whereas steady-state models have limited applicability. All the foregoing models that we have come across are examples of dynamic models. Continuous-time dynamic models necessarily involve derivatives to explain the rate of evolution. A first-principles (continuous) dynamic model was encountered in (2.6). Discrete-time dynamic models on the other hand are in the form of difference equations. Consequently, they rely on past outputs (and inputs) to capture the transients. Dynamic models thus have a memory. An empirical dynamic model was constructed for the liquid level system in Chapter 2. First-principles dynamic models are obtained by a suitable discretization of the continuous-time model. Note that steady-state models are a special case of dynamic models.

The focus of this text is to develop empirical dynamic models.

Lumped vs. distributed parameter models

In explaining the output variations, one could treat the output to be only a function of time or also address output variations in other dimensions. *Lumped* parameter models are based on the former assumption. The variations of output in other dimensions are neglected, rather lumped into a single quantity. *Distributed* parameter model, as the name suggests also examines the variations along other dimensions. The continuously stirred-tank reactor (CSTR) is usually given a lumped model treatment since it is safe to assume uniform concentration in the tank by virtue of the stirrer. A plug flow reactor (PFR) on the other hand calls for a distributed parameter model because the concentration of the product (and the unreacted component) varies along the length of the reactor in addition to time.

Lumped parameter models are easier to estimate because they are characterized by much fewer parameters compared to the distributed parameter model. Consequently, the data length and excitation requirements for estimation of these models are significantly different. Analogous to the case of time-invariant vs. time-varying systems, distributed parameter systems can also be modelled with the help of multiple (locally) lumped parameter models. Once again therefore it suffices to learn the theory of developing lumped parameter models.

Domain-based models

Finally, models can also be constructed in specific *domains*. The continuous-time and discrete-time are specific examples of this philosophy. In a broader sense, models are divided into those built in the raw domain and transform domain. The raw domain is the domain in which measurements are available to the user (typically the time domain). The transform domain model relates variables in a transformed space. The Fourier domain is a common domain in which transform models are built.

Example 3.9: Frequency Response Function

The output and input of the system described in (3.2) are related in the Fourier domain as

$$Y(\omega) = G(e^{j\omega})U(\omega) \tag{3.7}$$

where $Y(\omega)$ and $U(\omega)$ are the Fourier transforms of the output $y[k]$ and input $u[k]$, respectively. The function $G(e^{j\omega})$ is known as the frequency response function or more commonly as the transfer function (see Section 17.4.2).

The frequency response function is also the Fourier transform of the impulse response function $g[k]$.

Chapter 5 reviews the concepts of frequency-domain models for linear time-invariant systems. In several applications, measurements are directly available as a function of frequency. In such cases, the raw domain is the frequency domain itself.

Models can also be in mixed domains. An emerging example of this approach is model development in the wavelet domain, which allows the user to model the variations as a joint function of time and frequency (scale). Wavelet domain models have shown to be potentially elegant tools for modeling of multiscale and time-varying systems. Section 25.1.5 discusses models described in the wavelet domain for the purpose of modeling linear time-varying systems. In general, the choice of transform depends on the nature of the process characteristics under study.

Choosing the "correct" model

A key challenge in identification is to select the "correct" type of model. The fact is that the notion of a "right" model is only idealistic. We are practically interested in obtaining a useful and working model. Four primary factors govern its choice and development, namely,

i. **End-use**: This is a significant factor in determining the nature, structure and accuracy of the model to be developed. The entire identification exercise may be tailored to suit a particular application. Control-relevant identification is a paradigm based on this philosophy. Models meant to be used in control are expected to have good predictive abilities in only frequency regimes where control is effective. Moreover, the controller design and model development can be jointly performed.

ii. **Estimation considerations**: When two or more structures are equally likely candidates, the ease of estimation is a deciding factor. Auto-regressive models, for instance, result in linear predictors whereas moving average models produce non-linear predictors (see Chapters 9 and 18). Therefore, in time-series modeling, auto-regressive models are favored from an estimation complexity viewpoint. Linear predictors with least squares criteria produce unique parameter estimates. We had discussed this somewhat in detail in Chapter 2.

iii. **Model simplicity**: Given that any model is only an abstraction of the process, models that are simpler are always preferable. This is also the principle of *parsimony*. Parsimonious and simpler models are convenient from both estimation and implementation viewpoints.

iv. **Physical meaningfulness**: Several applications demand that models are physically meaningful, particularly that the model structure has some correspondence to the physics or biology of the process. Grey-box modeling places maximum importance to this criterion.

It would not be an overemphasis to reiterate the role of data quality in selecting the "correct" model. The final model is heavily leveraged on the inputs that are used to excite the system and the noise present in the system. Therefore, a preliminary examination of the data is always useful in obtaining an idea of the "best" model that one may be able to fit. For example, step response data cannot be expected to be used to develop highly accurate models.

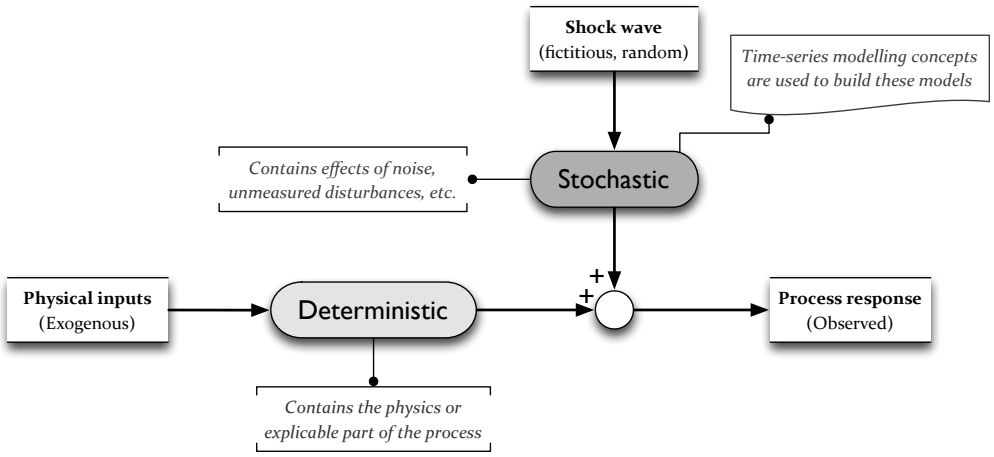


FIGURE 3.4 (**SEE COLOR INSERT**) Composite model from identification.

3.2.2 MODELS FOR IDENTIFICATION

From the preceding discussion, it is clear that a plethora of models are possible for a given system depending on the assumptions and characteristics we wish to describe. In this text, however, we shall focus on the development of *discrete-time, linear, time-invariant, dynamic, single-scale models*. To keep the description short, we refer to these models as linear time-invariant (LTI) models. At this point it is also clear that the overall model of interest is a deterministic-plus-stochastic model. Figure 3.4 shows a schematic of the overall model that we aim to develop.

An important advantage of choosing LTI descriptions is that a large class of stochastic effects can be also modeled as response of an LTI system driven by a fictitious random signal. This fictitious signal can be treated as a *shock wave* like signal as illustrated in Figure 3.4. Chapter 9 presents the theoretical details and derivations pertaining to the stochastic modeling.

The forthcoming Chapter 4 lays the foundations for mathematical descriptions of *discrete-time* LTI deterministic processes.

REVIEW QUESTIONS

R3.1. Define the term *model*.

R3.2. What is the basis for model classification?

R3.3. Identify three key merits of first-principles and empirical models.

R3.4. What classes of models does system identification generally result in?

R3.5. Describe the differences between a deterministic and a stochastic model.

R3.6. What are lumped parameter models? For what class of processes are they suited?

R3.7. What are the considerations for selecting a model class?

R3.8. Explain the justification for assuming linearity and time-invariance. Does this hold for all processes?

R3.9. Where does grey-box model get its name from?

R3.10. Identify the key difference between non-parametric and parametric models.

R3.11. What are transform domain models and how can they be useful?

EXERCISES

E3.1. Give two examples of distributed parameter models.

E3.2. Classify the systems (i) $y[k] = u[k^2]$ and (ii) $y[k] = sign(u[n-2])$ and (iii) $y[k] = \sin(u[k])$ as one of causal/non-causal, time-varying/time-invariant, static/dynamic and linear/non-linear.

E3.3. Processes are, in general, non-linear. Quite often we work with models that are linear approximations using a technique known as *linearization*, which constructs a first-order Taylor's series approximation of the non-linear model around a steady-state. A non-linear dynamic conical liquid-level system is governed by the differential equation,

$$\frac{dV(t)}{dt} = F_i(t) - C_v\sqrt{h(t)} \tag{3.8}$$

where $V(t) = \frac{1}{3}\pi r^2(t)h(t)$ is the volume of the liquid in the conical tank, C_v is the valve coefficient, $h(t)$ and $F_i(t)$ are the liquid level and inlet flow rate, respectively. Given the semi-angle $\theta = 30^{\text{deg}}$, steady-state inlet flow $F_{i,s} = 2$ l/min and $C_v = 0.8$ units, do the following:

a. Determine the steady-state level h_s.
b. Construct an approximate linear ODE for the deviation variable $\tilde{h}(t) = h(t) - h_s$ in terms of $\tilde{F}(t) = F_i(t) - F_s$ through the linearization of the non-linear ODE.
c. Using the approximate linear model, discuss how C_v and θ influence the liquid level response to a perturbation in the steady-state when $F_i(t)$ is held fixed at its steady-state.

E3.4. The linear ODE obtained in **E3.3.** is continuous-time in nature. Develop two different approximate discretized models, i.e., difference equation models, for the linear system using the techniques of numerical integration, namely, the Euler's backward and forward differencing methods.