

Wan Fook Ping

Test 2

$$Q1. a) i) G(s) = \frac{(s+3)}{(s+4)(s+2-j)(s+2+j)}$$

$$G(s) = \frac{(s+3)}{(s+4)(s^2+4s-5)} \quad \#$$

$$ii) \sigma_a = \frac{\sum \text{finite poles} - \sum \text{finite zeros}}{\# \text{ finite poles} - \# \text{ finite zeros}}$$

$$\sigma_a = \frac{(-4-2+j-2-j) - (-3)}{3-1}$$

$$\sigma_a = -2.5 \quad \#$$

$$\theta_a = \frac{(2k+1)\pi}{\# \text{ finite poles} - \# \text{ finite zeros}}$$

$$k=0, \theta_a = \frac{\pi}{3-1} = \frac{\pi}{2} \quad \#$$

$$k=1, \theta_a = \frac{3\pi}{3-1} = \frac{3}{2}\pi \quad \#$$

$$k=2, \theta_a = \frac{5\pi}{3-1} = \frac{5}{2}\pi = \frac{\pi}{2}$$

$$iii) \theta_{\text{depart } a} = \sum \angle \text{zeros to } A - \sum \angle \text{poles to } A + 180^\circ$$

$$\theta_{\text{depart } a} = \tan^{-1}\left(\frac{1}{-2-3}\right) - \left(90^\circ + \tan^{-1}\left(\frac{1}{-2-j}\right)\right) + 180^\circ$$

$$\theta_{\text{depart } a} = 95^\circ - (90^\circ + 26.57^\circ) + 180^\circ$$

$$\theta_{\text{depart } a} = 108.43^\circ$$

$$\theta_{\text{depart } b} = 360^\circ - 108.43^\circ$$

$$\theta_{\text{depart } b} = 251.57^\circ$$

$$\theta_{\text{arrive}} = 0^\circ$$

①

$$b) i) |real| = 3 \omega_n$$

$$|real| = 0.59 (4.08)$$

$$|real| = 2.4072$$

$$|img| = \omega_n \sqrt{1 - \zeta^2}$$

$$|img| = 4.08 \sqrt{1 - 0.59^2}$$

$$|img| = 3.2942$$

$$\text{dominant poles} = 2.4072 \pm 3.2942j$$

$$ii) K = \frac{1}{|G(s)H(s)|}$$

$$K = \frac{|(s+4)(s+2+j)(s+2-j)|}{|(s+3)|}$$

$$\text{Sub } s = 2.4072 + 3.2942j :$$

$$K = 10.9978$$

$$iii) CLTF = \frac{K(s+3)}{(s+4)(s+2+j)(s+2-j)}$$

$$1 + \frac{K(s+3)}{(s+4)(s+2+j)(s+2-j)}$$

$$= \frac{K(s+3)}{s^3 + 4s^2 + 5s + 4s^2 + (6s + 20 + ks + 3k)}$$

$$= \frac{10.9978(s+3)}{s^3 + 8s^2 + (21s + 70.9978s) + 20 + 3(10.9978)}$$

$$= \frac{34.7878(s+3)}{s^3 + 8s^2 + 31.9978s + 52.9778}$$

$$\text{Char eq: } s^3 + 8s^2 + 31.9978s + 52.9778 = 0$$

$$\text{calculator: } s = -3.118$$

$$r = \frac{-3.118}{-2.4072} = 1.32 < 5$$

second order approximation is not valid.

$$Q 1. b) iv) T_s = \frac{4}{|real|}$$

$$T_s = \frac{4}{2.4072}$$

$$T_s = 1.66175$$

$$T_p = \frac{\pi}{|Img|}$$

$$T_p = \frac{\pi}{3.2942}$$

$$T_p = 0.95375$$

$$K_p = \lim_{s \rightarrow 0} \frac{(10.99)(s+3)}{(s+4)(s+2-j)(s+2+j)}$$

$$= \lim_{s \rightarrow 0} \frac{10.99(s+3)}{(s+4)(s^2+4s+5)}$$

$$= \frac{10.99(3)}{4(5)}$$

$$= 1.6485$$

$$ess = \frac{1}{1+K_p}$$

$$ess = \frac{1}{1+1.6485}$$

$$ess = 0.3776 \#$$

3

Q2 a) Pb controller :

$$T_{s \text{ old}} = \frac{4}{|\text{real}|}$$

$$T_{s \text{ old}} = \frac{4}{0.88}$$

$$T_{s \text{ old}} = 4.5455 \text{ s}$$

$$T_{s \text{ new}} = \frac{3}{5} T_{s \text{ old}}$$

$$T_{s \text{ new}} = 2.727 \text{ s}$$

$$T_{s \text{ new}} = \frac{4}{|\text{real new}|}$$

$$|\text{real new}| = \frac{4}{T_{s \text{ new}}}$$

$$|\text{real new}| = 1.47$$

$$|\text{real new}| = \zeta \omega_n$$

$$\omega_n = \frac{|\text{real new}|}{\zeta}$$

$$\omega_n = \frac{1.47}{0.4559}$$

$$\omega_n = 3.224$$

$$\text{new dominant poles, } s_{d \text{ new}} = 1.4668 \pm j2.8636$$

$$\theta_1 = \tan^{-1} \left(\frac{2.87}{3 - 1.47} \right) = 61.94^\circ$$

$$\theta_2 = \tan^{-1} \left(\frac{2.87 - 1}{2 - 1.47} \right) = 74.18^\circ$$

$$\theta_3 = \tan^{-1} \left(\frac{2.87 + 1}{2 - 1.47} \right) = 82.20^\circ$$

$$\theta_4 = 180^\circ - \tan^{-1} \left(\frac{2.87 - 2.65}{1.47 + 1} \right) = 174.91^\circ$$

$$\theta_5 = 180^\circ - \tan^{-1} \left(\frac{2.87 + 2.65}{1.47 + 1} \right) = 114.17^\circ$$

$$\zeta = \frac{-\ln \left(\frac{20}{100} \right)}{\sqrt{\pi^2 + \ln^2 \left(\frac{20}{100} \right)}}$$

$$\zeta = 0.4559$$

$$\zeta = \cos \theta$$

$$\theta = \cos^{-1}(0.4559)$$

$$\theta = 62.877^\circ$$

$$|\text{img}| = \omega_n \sqrt{1 - \zeta^2}$$

$$|\text{img}| = 3.224 \sqrt{1 - 0.4559^2}$$

$$|\text{img}| = 2.87$$

$$\begin{aligned}\theta_2 &= (61.99^\circ + 74.18^\circ + 82.20^\circ) - (174.91^\circ + 94.87^\circ) - 180^\circ \\ &= -231.46^\circ \\ &= 128.54^\circ\end{aligned}$$

⑤

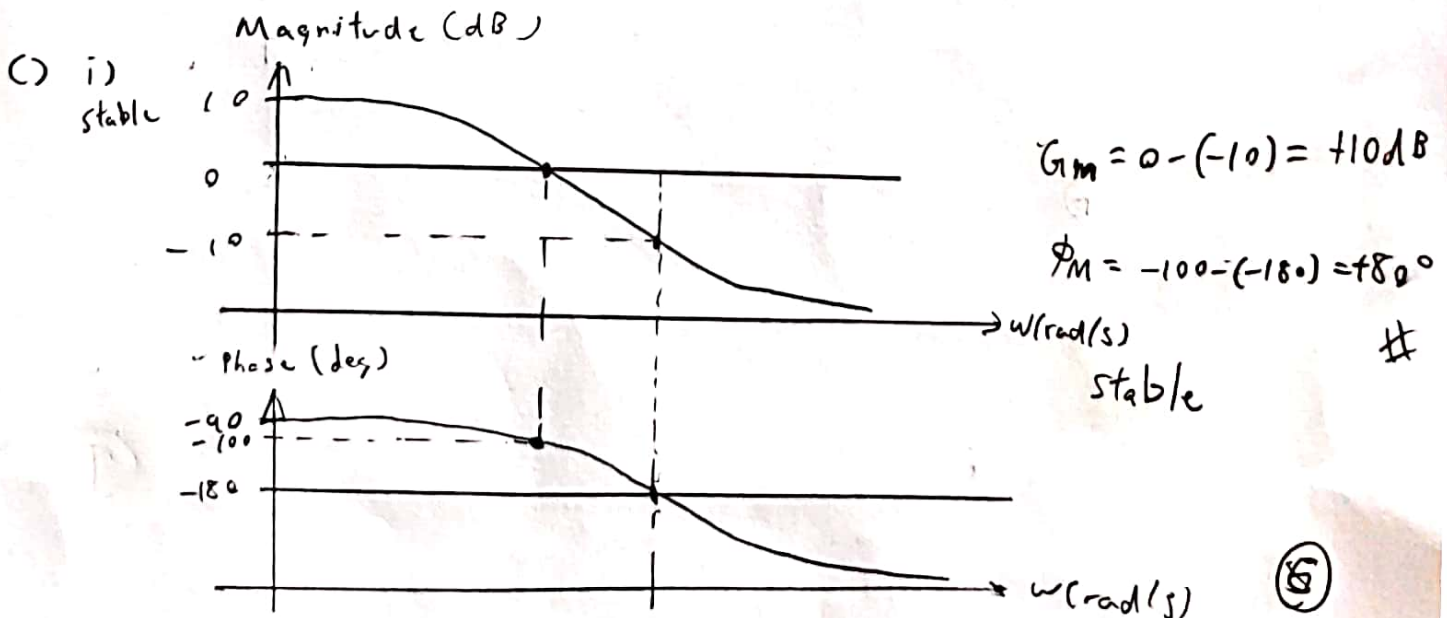
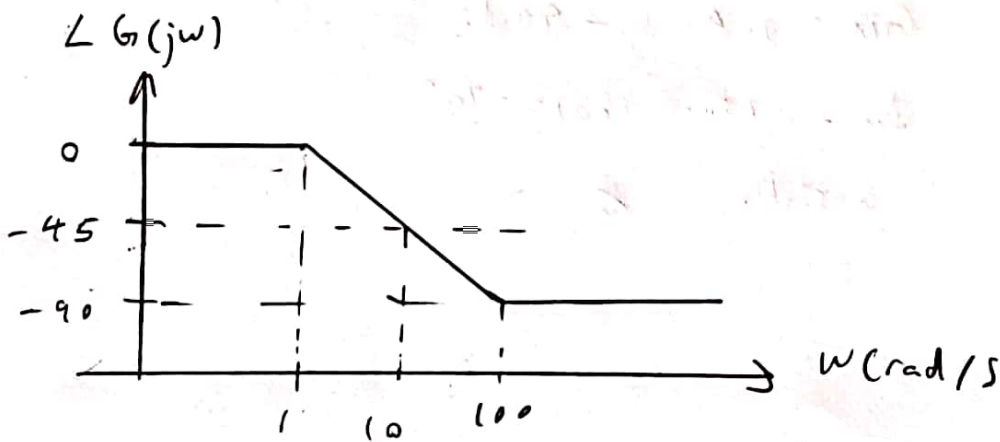
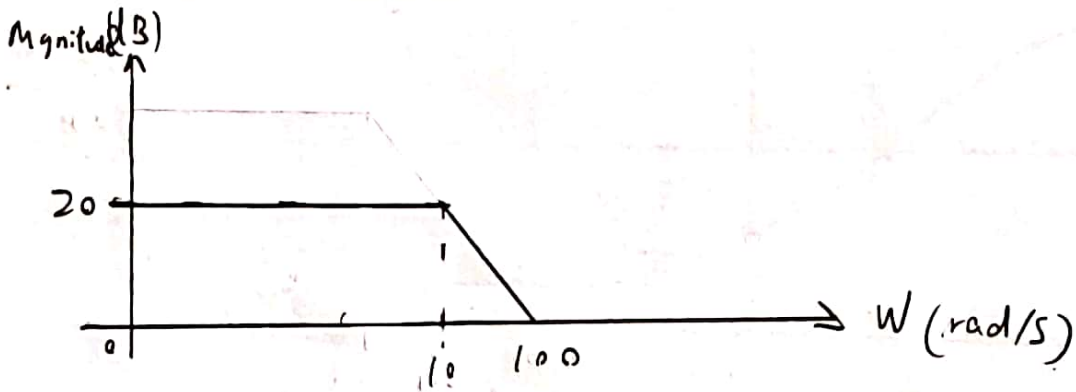
$$3 \text{ a) i) } G(s)_{dB} = 20 \log |2| - 20 \log |j\omega| - 20 \log |j\omega + 6|$$

$$= 21.58 - 20 \log |\omega| - 20 \log |j\omega + 6|$$

$$\text{ii) } G(20j)_{dB} = 21.58 - 20 \log |20| - 20 \log |j20 + 6|$$

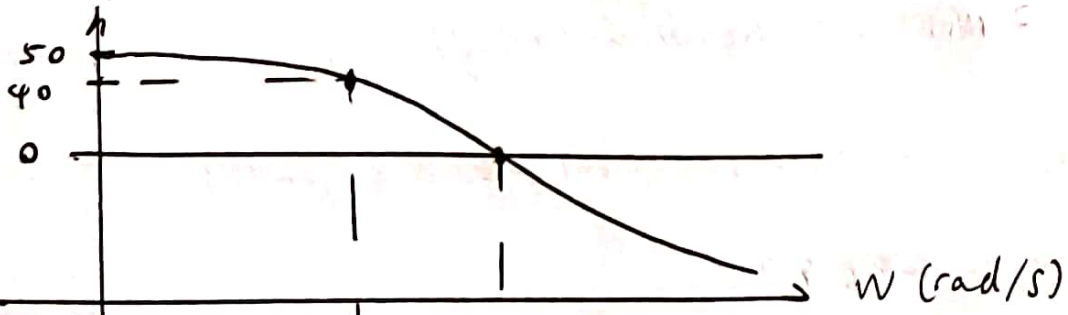
$$G(20j)_{dB} = -30.84$$

$$\text{b) } 20 \log(100) = 40 \text{ dB}$$

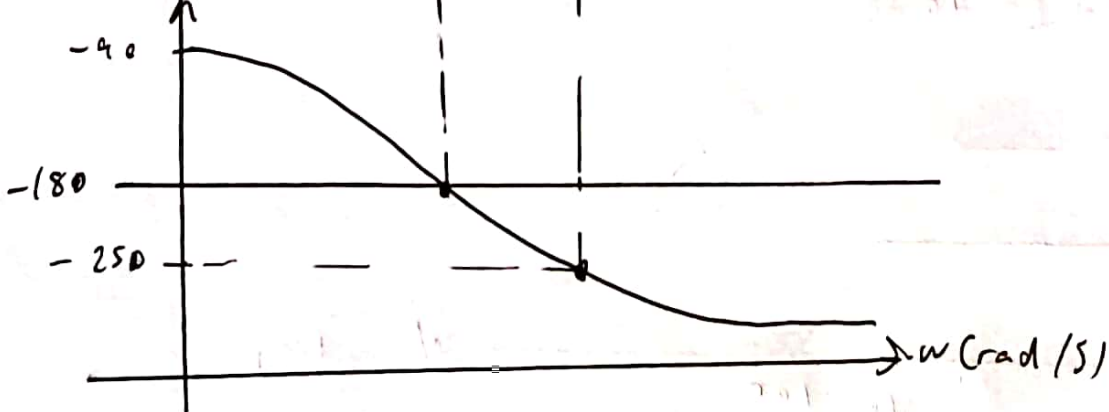


c) ii) Unstable

Magnitude



Phase (deg)



$$GM = 0 - 40 = -40 \text{ dB}$$

$$\phi_M = -250 - (-180) = -70^\circ$$

unstable #

⑦