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A17KE0297
SKEE3143 Sec 06
Test1
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Q1a) i) $G(s) = \dfrac{K(s+3)}{(s+4)(s+2+j)(s+2-j)}$

ii) $\sigma_a = \dfrac{\sum \text{finite poles} - \sum \text{finite zeroes}}{\#\text{ poles} - \#\text{ zeroes}}$

$= \dfrac{(-4-2-2)-(-3)}{3-1}$

$= -5/2 = -2.50$

$\theta_a = \dfrac{(2k+1)180^\circ}{\#\text{ poles} - \#\text{ zeroes}}$; $k = 0, 1, 2, \ldots$

$= \dfrac{(2k+1)(180^\circ)}{2}$

$= (2k+1)(90^\circ)$

$k=0,\ \theta_a = 90^\circ$

$k=1,\ \theta_a = 270^\circ$

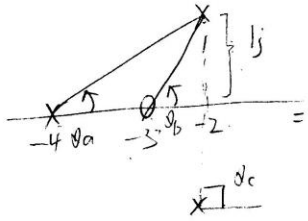
$k=2,\ \theta_a = 450^\circ \simeq 90^\circ$

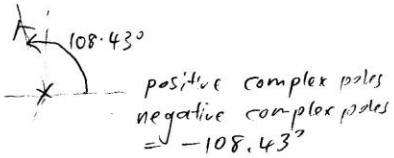
iii) angles of depature ; on complex poles.

$\sum \text{zero angles to A} - \sum \text{pole angle to A} + 180^\circ$

$= -\tan^{-1}(1/2) + \tan^{-1}(1/1) - 90^\circ + 180^\circ$

$= 108.43^\circ$





angle of arrival ; none ; no complex zeroes

b) OS% = 10%

$\zeta = 0.59$

$\omega_n = 4.08 \text{ rad/s}$

$\theta_d = \cos^{-1}(\zeta)$

$\theta_d = \cos^{-1}(0.59) = 53.843^\circ$

$S_{1,2} = -2.4 \pm 3.3j$ take from figure Q1.2 dominant

or $|Re| = \zeta\omega_n = 0.59(4.08) = 2.407$

$|Im| = \omega_n\sqrt{1-\zeta^2} = 4.08\sqrt{1-0.59^2} = 3.29$

ii) operational gain — RW table

$$G(s) = \frac{k(s+3)}{(s+4)(s+2+j)(s+2-j)} \quad \text{OLTF}$$

$$G(s) = \frac{k(s+3)}{(s+4)(s+2+j)(s+2-j) + k(s+3)} \quad \text{CLTF}$$

$$= \frac{k(s+3)}{s^3 + 8s^2 + (21+k)s + 20+3k}$$

$(s+2+j)(s+2-j)$

$= s^2 + (2-j)s + (2+j)s + 5$

$= s^2 + 4s + 5$

$(s+4)(s^2+4s+5)$

$= s^3 + 4s^2 + 5s + 4s^2 + 16s + 20$

$= s^3 + 8s^2 + 21s + 20$

s^3	1	$(21+k)$	0
s^2	8	$20+3k$	0
s^1	$-\frac{\begin{vmatrix} 1 & 21+k \\ 8 & 20+3k \end{vmatrix}}{8}$		0
s^0	$-\frac{\begin{vmatrix} 8 & 20+3k \\ 29.6+0.625k & 0 \end{vmatrix}}{29.6+0.625k}$		

$s_{1,2} = -2.4 \pm 3.3j$

$k = \frac{\text{Π pole length}}{\text{Π zero length}}$

$= \frac{\left(\sqrt{0.4^2+2.3^2}\right)\left(\sqrt{0.4^2+4.3^2}\right)}{\left(\sqrt{1.6^2+3.3^2}\right)\left(\sqrt{0.6^2+3.3^2}\right)}$

$= 36.974 / 3.354$

$= 11.024$

$s^1 = \frac{168+8k-20-3k}{8}$

$= \frac{148+5k}{8}$

$= 29.6 + 0.625k$

$s^0 = 20+3k$

$s^3 + 8s^2 + (21+11.024)s + 20+3(11.024) = 0$

$s^3 + 8s^2 + 32.024s + 53.072 = 0$

$s_1 = -3.178$

$s_2 = -2.41 \pm 3.3j$

iii) Ratio $= \frac{\text{minor pole dist}}{\text{major pole dist}}$

$= \frac{-3.178}{-2.41} = 1.32 < 5$; invalid to use 2nd order approximation.

iv)

$T_s = \frac{4}{|Re|} = \frac{4}{2.41} = 1.66\,s$

$T_p = \frac{\pi}{|Im|} = \frac{\pi}{3.3} = 0.95\,s$

Q2)

$Sd_{1,2} = -0.88 \pm j\,1.72$

$\%OS = 20\%$

$\zeta = \frac{-\ln(0.20)}{\sqrt{\pi^2 + (\ln(0.20))^2}} = 0.46$

$T_{s\,(old)} = \frac{4}{|Re|} = \frac{4}{0.88} = 4.55\,s$

$T_{s\,(new)} = \frac{3}{5}(4.55) = 2.73\,s$

overshoot unchanged, ζ unchanged

$2.73 = \frac{4}{|Re|_{new}}$; $|Re|_{new} = 1.47 = \zeta\omega_{n\,new}$

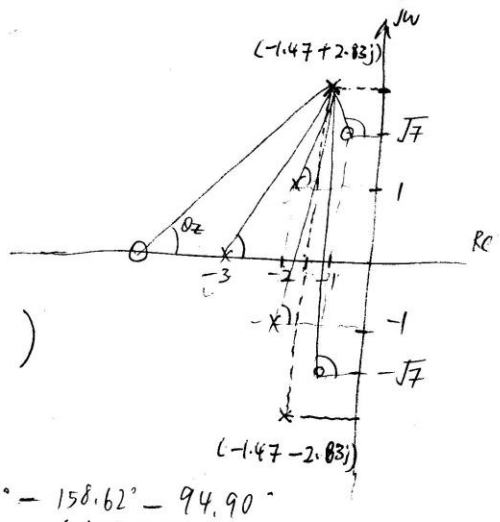
$\omega_{n\,new} = 3.188\ rad/s$

$|Im| = \omega_n\sqrt{1-(0.46)^2} = 2.83$

$Sd_{new} = -1.47 \pm 2.83j$

$180° = -\theta_z + \tan^{-1}\left(\frac{2.83}{1.53}\right) + \tan^{-1}\left(\frac{1.83}{0.53}\right) + \tan^{-1}\left(\frac{3.83}{0.53}\right) = (180° - \tan^{-1}\left(\frac{0.184}{0.47}\right))$

$-(180° - \tan^{-1}\left(\frac{5.48}{0.47}\right))$



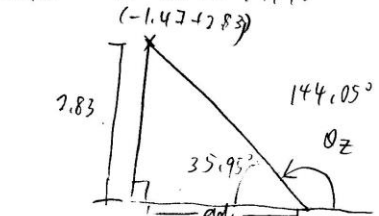
$180° = -\theta_z + 61.6 + 73.85 + 82.12° - 158.62° - 94.90°$

$180° = -\theta_z - 35.95°$

$\theta_z = -215.95° \simeq 144.05°$

$z = -1.47 + 3.90$

$= 2.43$



$\tan(35.95°) = 2.83/adj$; $adj = 3.90$

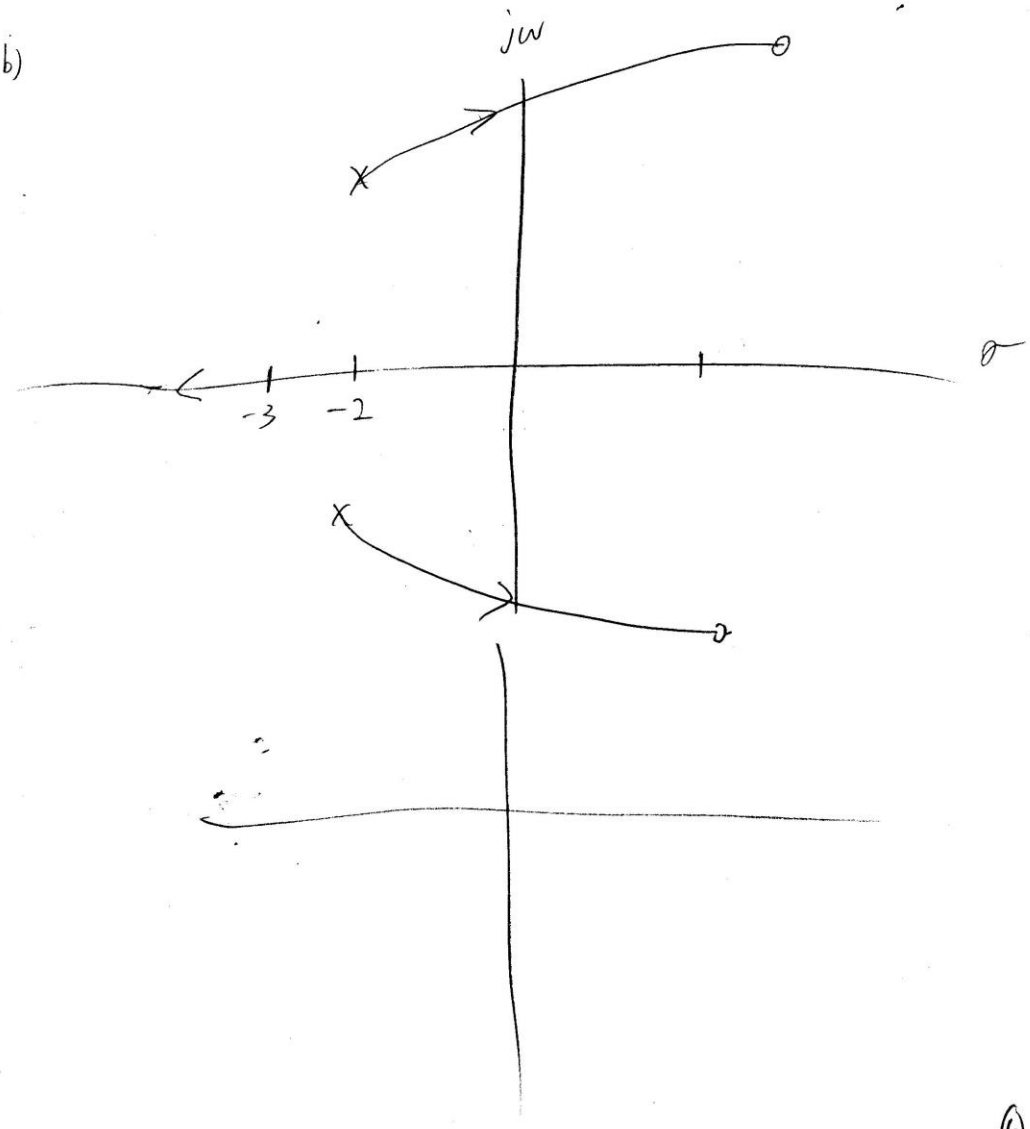
(3)

$K = \frac{\Pi \text{ pole length}}{\Pi \text{ zero length}}$ to $s = -1.47 \pm 2.83j$

$$= \frac{\left(\sqrt{2.83^2 + 1.53^2}\right)\left(\sqrt{1.83^2 + 0.53^2}\right)\left(\sqrt{3.83^2 + 0.53^2}\right)}{\left(\sqrt{0.47^2 + 0.184^2}\right)\left(\sqrt{0.47^2 + 5.48^2}\right)\left(\sqrt{2.9^2 + 2.83^2}\right)}$$

$=$

b)



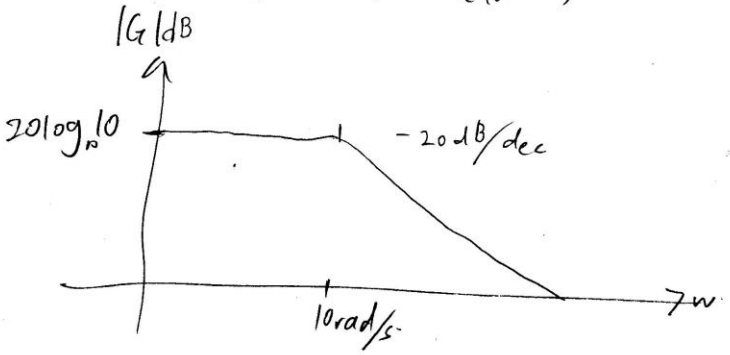
Q3 a) i) $G(s) = \frac{12}{s(s+6)}$ = $20\log(w) \times 20\log\left(\frac{1}{\sqrt{\left(\frac{w}{6}\right)^2+1}}\right) \times 20\log\frac{12}{6}$

ii) $w = 20\ rad/s$

$|G(jw)| = 20\log 2 \times 20\log 20 \times 20\log\left(\frac{1}{\sqrt{\left(\frac{20}{6}\right)^2+1}}\right)$

$= -1696.9\ dB.$

b) $G(s) = \frac{100}{(s+10)} = \frac{100/10}{\left(\frac{s}{10}+1\right)} = \frac{10}{\left(\frac{s}{10}+1\right)}$



c) i)