

Q1

$$a) G(s) = \frac{K(s+3)}{(s+4)(s-2+j)(s-2-j)}$$

$$= \frac{K(s+3)}{(s+4)(s^2+4s+5)}$$

ii asymptote point

$$\sigma_a = \frac{-4-2-2+3}{3-1}$$

$$= -\frac{5}{2}$$

$$\theta_a = \frac{(2k+1)\pi}{3-1}$$

$$= \frac{(2k+1)\pi}{2}$$

$$k=0, 1, 2$$

$$\theta_a = \frac{\pi}{2} = 90^\circ$$

$$\theta_a = \frac{3\pi}{2} = 270^\circ$$

$$\theta_a = \frac{5\pi}{2} = 450^\circ$$

angled departure

$$\theta = \tan^{-1}(1) - [\tan^{-1}(\frac{1}{2} + 90)] + 180^\circ$$

$$= 108.43^\circ$$

(b) $\zeta = 10\%$ $\zeta = 0.59$ $\omega_n = 4.08 \text{ rad/s}$

(i)

$$\begin{aligned} |\text{real part}| &= \zeta \omega_n \\ &= 0.59 \times 4.08 \\ &= 2.4072 \end{aligned}$$

$$\begin{aligned} |\text{imaginary part}| &= \omega_n \sqrt{1 - \zeta^2} \\ &= 4.08 \sqrt{1 - (0.59)^2} \\ &= 3.294 \end{aligned}$$

\therefore dominant pole = $-2.4072 \pm 3.294j$

(ii)

$$\begin{aligned} K &= \frac{1}{|G(s)|} \\ &= \frac{(s+4) |s-2+j| |s-2-j|}{|s+3|} \quad \text{substitute } s = -2.4072 + 3.294j \\ &= 8.1279 \end{aligned}$$

(iii)

$$\text{ratio} = \frac{|-4|}{|-2.4072|} = 1.66 < 5$$

\therefore the second order approximation is not valid.

(iv)

$$T_s = \frac{4}{|\text{real}|} = \frac{4}{2.4072} = 1.66 \text{ s}$$

$$T_p = \frac{\pi}{|\text{imag}|} = \frac{\pi}{3.294} = 0.9537 \text{ s}$$

$$\begin{aligned} K_p &= \lim_{s \rightarrow 0} G(s) \\ &= \lim_{s \rightarrow 0} \frac{K(s+3)}{(s+4)(s-2+j)(s-2-j)} \\ &= \frac{8.1279(3)}{(4)(-2+j)(-2-j)} \\ &= 1.2192 \end{aligned}$$

$$\begin{aligned} e_{ss} &= \frac{1}{1+K_p} = \frac{1}{1+1.2192} \\ &= 0.4506 \neq \end{aligned}$$

Q2

$$(a) T_{old} = \frac{4}{|\text{real}|} = \frac{4}{0.88} = 4.55 \text{ s} \quad \%OS = 20\% \quad \delta = \frac{-\ln\left(\frac{20}{100}\right)}{\sqrt{\pi^2 + \ln^2\left(\frac{20}{100}\right)}}$$

$$T_{new} = \frac{4}{|\text{real}_{new}|} = 4.55 \times \frac{3}{5}$$

$$= 0.4559$$

$$|\text{real}_{new}| = \underline{1.4652}$$

$$\tan^{-1}\left(\frac{\text{imag}_{new}}{\text{real}_{new}}\right) = \cos^{-1}(0.4559)$$

$$\tan^{-1}\left(\frac{\text{imag}}{1.4652}\right) = 62.88^\circ$$

$$\text{imag}_{new} = 2.8604$$

$$G_{PD} = (s+z)$$

$$\text{dominant poles} = -1.4652 \pm 2.8604j$$

$$G_{PD} G = \frac{k(s+z)(s^2-2s+8)}{(s+3)(s^2+4s+5)}$$

$$\cancel{Ls+z} = \cancel{L}$$

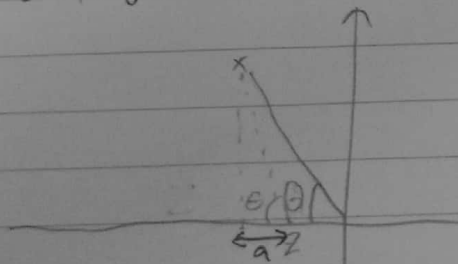
$$\angle s+z + \angle s^2-2s+8 - [\angle s+3 + \angle s^2+4s+5] = 180^\circ$$

$$\text{sub } s = -1.4652 + 2.8604j$$

$$\angle s+z + (-70.86) - [61.78^\circ + 156.07] = 180^\circ$$

$$\angle s+z = 468.71$$

$$= 108.71^\circ$$



$$\tan(180 - 108.71) = \frac{2.8604}{a}$$

$$a = 0.9687$$

$$\approx 0.97$$

$$z = -(1.4652 + 0.97)$$

$$= -0.4952$$

$$\underline{z = -0.5}$$

P.D
controller



$$\therefore G_{PD} = \cancel{(s+0.5)}$$

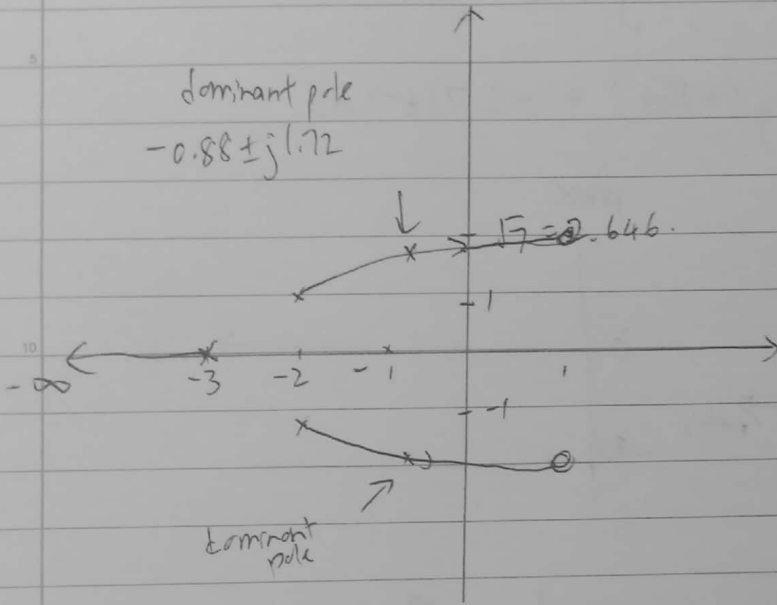
$$G_{PD} = (s+0.5)$$

Q2 (b)

uncompensated

Zeros = $1 \pm \sqrt{5}i$

poles = $-3, -2+i, -2-i$

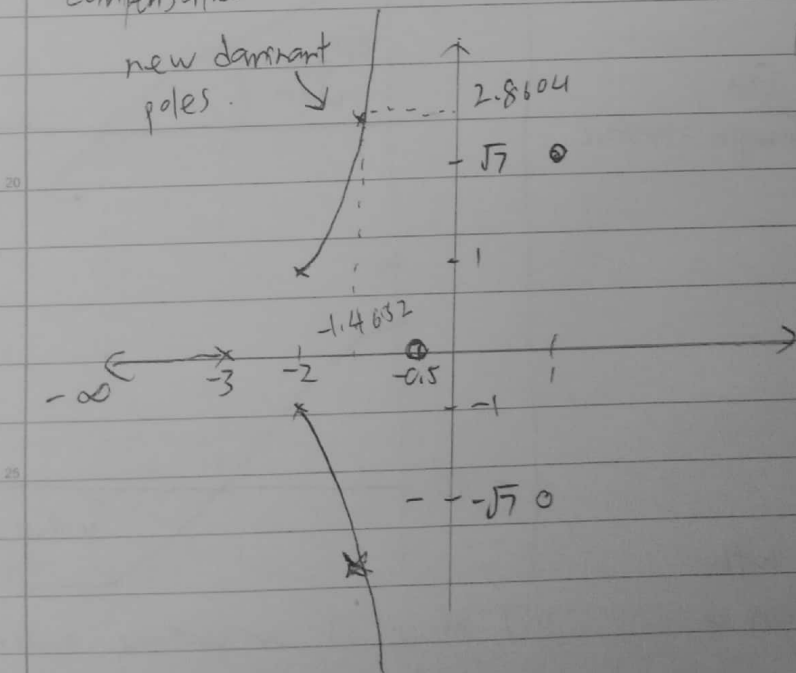


compensated

new dominant poles

zero = $1 \pm \sqrt{5}i, -0.5$

poles = $-3, -2+i, -2-i$

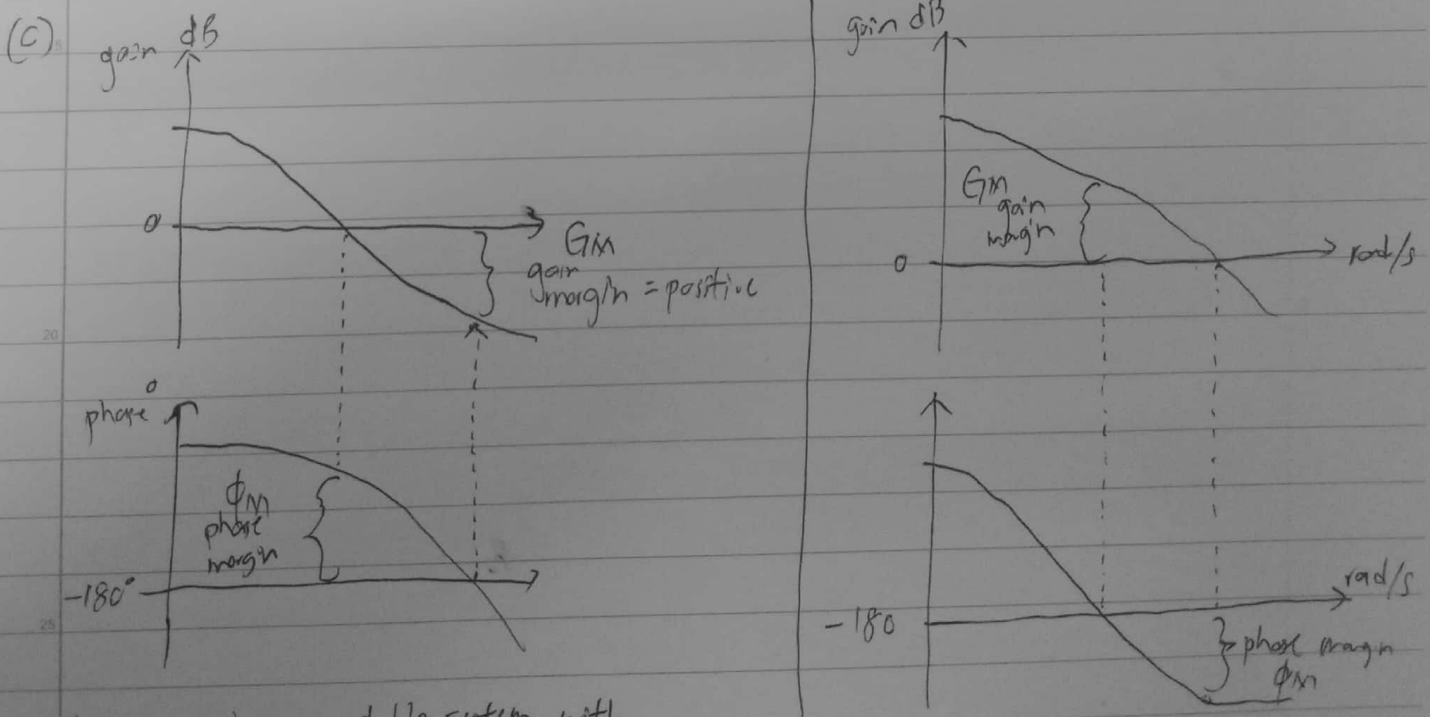
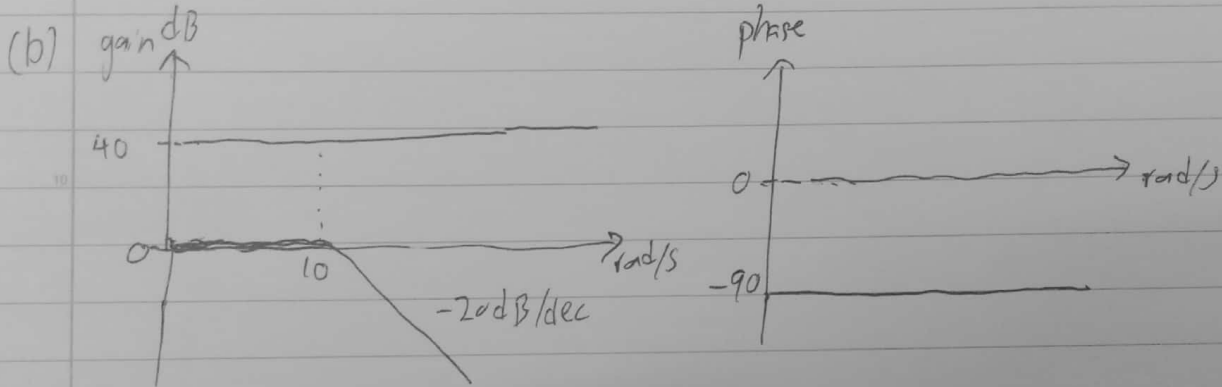


Q3

(a) (i) $20 \log 12 - 20 \log |j\omega| - 20 \log |j\omega + 6|$

(ii) $20 \log 12 - 20 \log |j20| - 20 \log |j20 + 6|$

$20 \log 12 - 26.02 \text{ dB} = 20 \log (2\sqrt{109}) = -2.7167 \text{ dB}$



(i) above is a stable system with phase margin and gain margin a positive value

(ii) above is an unstable system which both value of negative values