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Test 2

Q1. a) i) $$G(s) = \frac{(s+3)}{(s+4)(s+2-j)(s+2+j)}$$

$$G(s) = \frac{(s+3)}{(s+4)(s^2+4s-5)}$$ #

ii) $$\sigma_a = \frac{\Sigma \text{ finite poles} - \Sigma \text{ finite zeros}}{\# \text{ Finite poles} - \# \text{ finite zeros}}$$

$$\sigma_a = \frac{(-4-2+j-2-j)-(-3)}{3-1}$$

$$\sigma_a = -2.5$$ #

$$\theta_a = \frac{(2k+1)\pi}{\# \text{ finite poles} - \# \text{ finite zeros}}$$

$k=0$, $\theta_a = \frac{\pi}{3-1} = \frac{\pi}{2}$ #

$k=1$, $\theta_a = \frac{3\pi}{3-1} = \frac{3}{2}\pi$ #

$k=2$, $\theta_a = \frac{5\pi}{3-1} = \frac{5}{2}\pi = \frac{\pi}{2}$

iii) $\theta_{depart\,a} = \Sigma \angle \text{zeros to A} - \Sigma \angle \text{poles to A} + 180°$

$\theta_{depart\,a} = \tan^{-1}\left(\frac{1}{-2-3}\right) - \left(90° + \tan^{-1}\left(\frac{1}{-2-4}\right)\right) + 180°$

$\theta_{depart\,a} = 45° - (90° + 26.57) + 180°$

$\theta_{depart\,a} = 108.43°$

$\theta_{depart\,b} = 360° - 108.43°$

$\theta_{depart\,b} = 251.57°$

$\theta_{arrive} = 0°$

0

Scanned with CamScanner

b) i) $|real| = \zeta w_n$

$|real| = 0.59\,(4.08)$

$|real| = 2.4072$

$|img| = w_n \sqrt{1-\zeta^2}$

$|img| = 4.08\sqrt{1-0.59^2}$

$|img| = 3.2942$

dominant poles $= 2.4072 \pm 3.2942j$

ii) $$K = \frac{1}{|G(s)H(s)|}$$

$$K = \frac{|(s+4)||(s+2+j)||(s+2-j)|}{|(s+3)|}$$

Sub $s = 2.4072 + 3.2942j$:

$K = 10.99$

iii) $$CLTF = \frac{\dfrac{K(s+3)}{(s+4)(s+2+j)(s+2-j)}}{1+\dfrac{K(s+3)}{(s+4)(s+2+j)(s+2-j)}}$$

$$= \frac{K(s+3)}{s^3 + 4s^2 + 5s + 4s^2 + 16s + 20 + Ks + 3K}$$

$$= 10.99\,(s+3)$$

Q1. b) iv) $T_s = \frac{4}{|real|}$ $T_p = \frac{\pi}{|Img|}$

$T_s = \frac{4}{2.4072}$ $T_p = \frac{\pi}{3.2942}$

$T_s = 1.6617s$ $T_p = 0.9537s$

$$K_p = \lim_{s \to 0} \frac{(10.99)(s+3)}{(s+4)(s+2-j)(s+2+j)}$$

$$= \lim_{s \to 0} \frac{10.99(s+3)}{(s+4)(s^2+4s+5)}$$

$$= \frac{10.99(3)}{4(5)}$$

$$= 1.6485$$

$$e_{ss} = \frac{1}{1+K_p}$$

$$e_{ss} = \frac{1}{1+1.6485}$$

$$e_{ss} = 0.3776$$ #

(3)

Q2. a) PD controller:

$$T_{s\,old} = \frac{4}{|real|}$$

$$T_{s\,old} = \frac{4}{0.88}$$

$$T_{s\,old} = 4.5455\,s$$

$$T_{s\,new} = \frac{3}{5} T_{s\,old}$$

$$T_{s\,new} = 2.727\,s$$

$$T_{s\,new} = \frac{4}{|real\,new|}$$

$$|real\,new| = \frac{4}{T_{s\,new}}$$

$$|real\,new| = 1.47$$

$$|real\,new| = \zeta \omega_n$$

$$\omega_n = \frac{|real\,new|}{\zeta}$$

$$\omega_n = \frac{1.47}{0.4559}$$

$$\omega_n = 3.224$$

$$\zeta = \frac{-\ln\left(\frac{20}{100}\right)}{\sqrt{\pi^2 + \ln^2\left(\frac{20}{100}\right)}}$$

$$\zeta = 0.4559$$

$$\zeta = \cos\theta$$

$$\theta = \cos^{-1}(0.4559)$$

$$\theta = 62.877^\circ$$

$$|img| = \omega_n\sqrt{1-\zeta^2}$$

$$|img| = 3.224\sqrt{1-0.4559^2}$$

$$|img| = 2.87$$

new dominant poles, $s_{d\,new} = 1.4668 \pm j2.8636$

$$\theta_1 = \tan^{-1}\left(\frac{2.87}{3-1.47}\right) \quad 61.94^\circ$$

$$\theta_2 = \tan^{-1}\left(\frac{2.87-1}{2-1.47}\right) = 74.18^\circ$$

$$\theta_3 = \tan^{-1}\left(\frac{2.87+1}{2-1.47}\right) = 82.20^\circ$$

$$\theta_4 = 180^\circ - \tan^{-1}\left(\frac{2.87-2.65}{1.47+1}\right) = 174.91^\circ$$

$$\theta_5 = 180^\circ - \tan^{-1}\left(\frac{2.87+2.65}{1.47+1}\right) = 114.17^\circ$$

$$\theta_2 = (61.94^\circ + 74.18^\circ + 82.20^\circ) - (174.91^\circ + 94.87^\circ) - 180^\circ$$

$$= -231.46^\circ$$

$$= 128.54^\circ$$

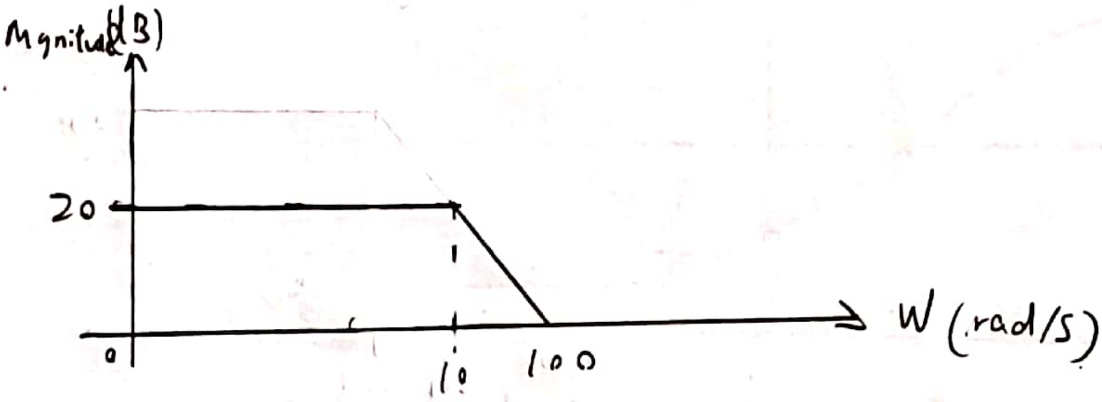
3 a) i) $G(s)_{dB} = 20\log|2 - 20\log|jw| - 20\log|jw+6|$

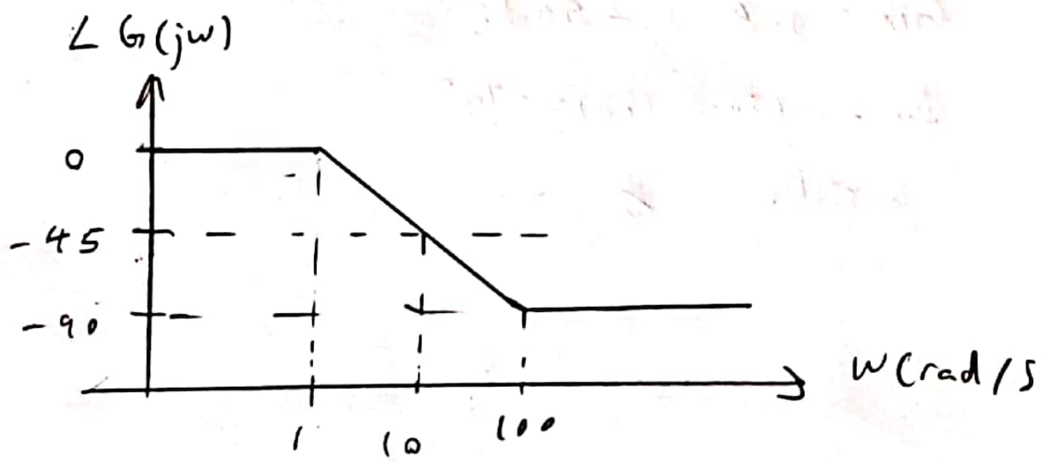
$= 21.58 - 20\log|W| - 20\log|jw+6|$

ii) $G(20j)_{dB} = 21.58 - 20\log|20| - 20\log|j20+6|$

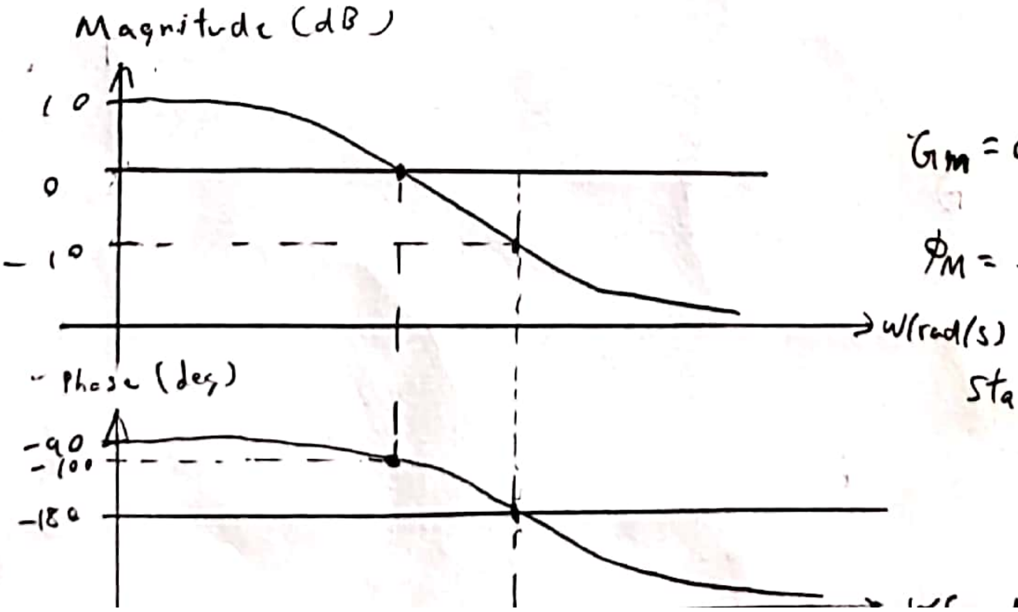
$G(20j)_{dB} = -30.84$

b) $20\log(100) = 40\ dB$





c) i) stable



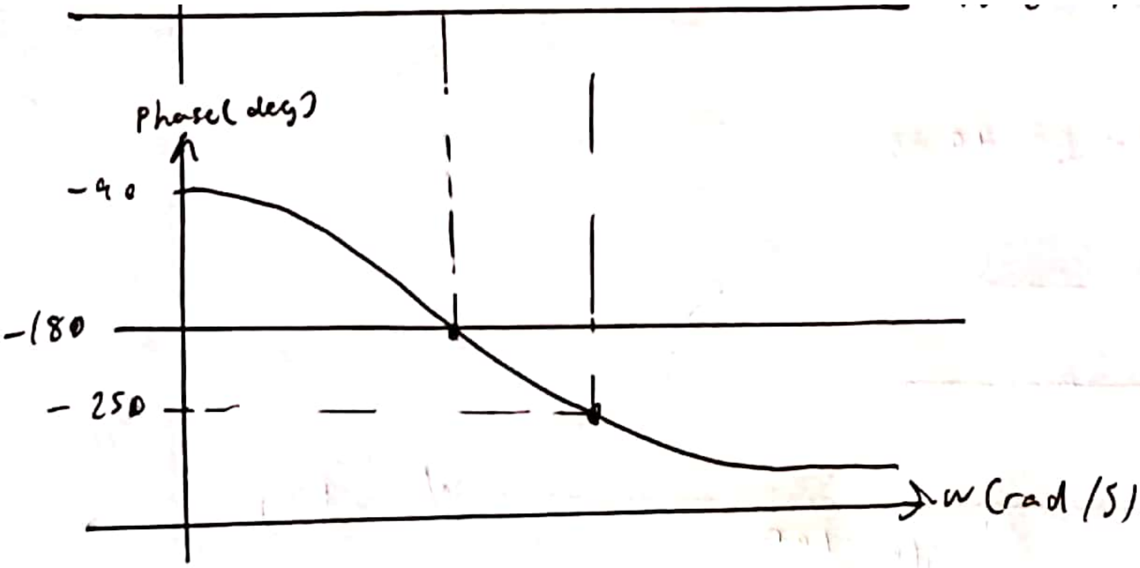
$G_m = 0 - (-10) = +10\ dB$

$\phi_M = -100 - (-180) = +80°$

#

stable

6



$G_M = 0 - 40 = -40\,dB$

$\phi_M = -250 - (-180) = -70^\circ$

unstable #