

Test 1 Control

Q1.

(a) (i) $G(s) = \frac{K(s+3)}{(s+4)(s+2+j1)(s+2-j1)}$

(ii) $\sigma_a = \frac{\sum \text{poles} - \sum \text{zeros}}{\# \text{ pole} - \# \text{ zeros}} = \frac{8 - 3}{3 - 1} = 2.5$

$\theta_a = \frac{(2k+1)\pi}{\# \text{ pole} - \# \text{ zero}} = \frac{(2+1)\pi}{3-1} = 1.57$

, $k=0, \pm$



(iii) angle of departure;

$\theta_a = \sum (\theta_{\text{pole to B}})$

(b) $\%OS = 10\%$, $\zeta = 0.59$, $\omega_n = 4.08 \text{ rad/s}$

(i) CL poles

$\theta = \cos^{-1}(\zeta)$
 $= \cos^{-1}(0.59)$
 $= 53.84^\circ$ — draw to the graph.

$-2.4 + j3.2$

$s = \text{CL poles (approximate)} = -2.4 + j3.2$

$G_{LTF} = \frac{K(s+3)}{(s+4)(s+2+j1)(s+2-j1)(s+3)}$

(ii) use magnitude properties; $|G(s)| = 1$

$\left| \frac{K(s+3)}{(s+4)(s+2+j1)(s+2-j1)} \right| = 1$

at $s = -2.4 + j3.2$

$\frac{K(-2.4 + j3.2 + 3)}{(-2.4 + j3.2 + 4)(-2.4 + j3.2 + 2 + j1)(-2.4 + j3.2 + 2 - j1)} = 1 \rightarrow K = 10.37$

Q1

(b)
(iii)
$$G(s) = \frac{10.37(s+3)}{(s+4)(s+2+j)(s+2-j)}$$

second order approximation is not valid; because the third poles $s = -4$ and the other poles at $s = -2 \pm j$, ratio between third pole and dominant pole is less than 5

$$\text{ratio} = \frac{4}{2} = 2 \neq 5$$

(iv)
$$T_s = \frac{4}{|\text{real}|} = \frac{4}{\zeta \omega_n}$$

$$= \frac{4}{0.59(4.08)}$$

$$= 1.66 \text{ s}$$

$$T_p = \frac{\pi}{|\text{imag}|} = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}}$$

$$= \frac{\pi}{4.08(\sqrt{1-0.59^2})}$$

$$= 0.95 \text{ s}$$

ess

Q2

$$G(s) = \frac{K(s^2 - 2s + 8)}{(s+3)(s^2 + 4s + 5)}$$

$$s_d = -0.88 \pm j1.72$$

$$\%OS = 20\%$$

(c) $T_{snew} = \frac{3}{5} T_s$

$$\%OS = 20\% \rightarrow \zeta = 0.455$$

~~from~~

from $s_d = -0.88 \pm j1.72$

$$-0.88 = -\zeta \omega_n$$

$$-0.88 = -0.455 \omega_n$$

$$\omega_n = 1.93$$

$$\text{imag} = j1.72 = j\omega_n \sqrt{1-\zeta^2}$$

$$T_p = \frac{\pi}{\omega_d} = \frac{\pi}{1.72} = 1.83s, \quad T_s = \frac{4}{|\text{real}|} = \frac{4}{0.88} = 4.54s$$

(desired)

$$T_{pnew} = \frac{3}{5} T_p$$

$$= \frac{3}{5} (1.83)$$

$$= 1.098s$$

$$1.098 = \frac{\pi}{\omega_d} \rightarrow \omega_d = 2.86$$

$$\omega_d = \omega_n \sqrt{1-\zeta^2}$$

$$\omega_n = 3.21 \text{ rad/s}$$

$$\text{CL-poles (desired)} = -\zeta \omega_n \pm j\omega_d$$

$$= -1.46 \pm j2.86$$

$$\sum \theta_p - \sum \theta_z = 180^\circ$$

$$\theta_4 + \theta_3 + \theta_6 - \theta_1 - \theta_2 - \theta_5 = 180$$

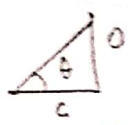
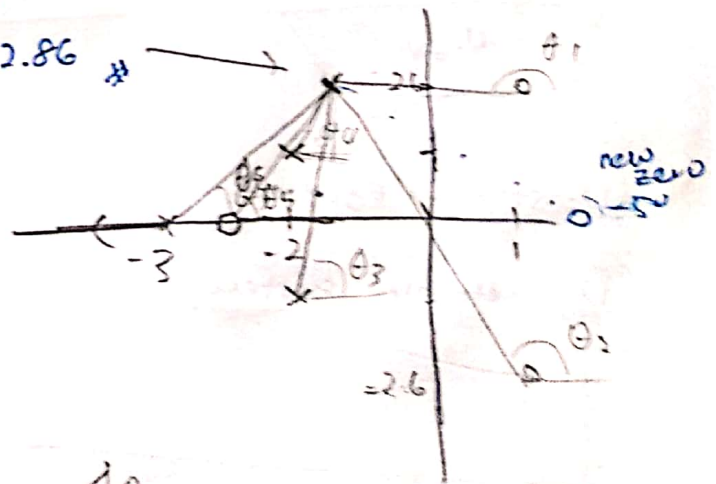
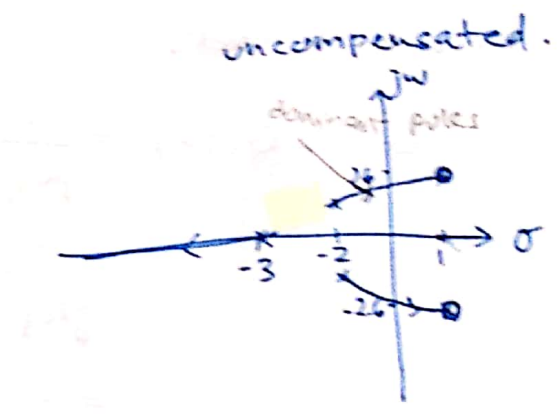
$$\theta_5 = 250.04^\circ$$

$$\alpha_1 = \tan^{-1}\left(\frac{0.26}{3}\right) = 4.95$$

$$\theta_1 = 180 - 4.95 = 175.0$$

$$\theta_2 = 180 - \tan^{-1}\left(\frac{4.546}{2.46}\right) = 114.2$$

$$\theta_6 = \tan^{-1}\left(\frac{2.86}{1.54}\right) = 61.69$$



$$\theta_4 = \tan^{-1}\left(\frac{1.86}{0.54}\right) = 73.81^\circ$$

$$\theta_3 = \tan^{-1}\left(\frac{4.86}{0.54}\right) = 83.66^\circ$$

new zero at positive side.

Q3. $G(s) = \frac{12}{s(s+6)}$

(a) (i)

(ii) when $\omega = 20 \text{ rad/s}$

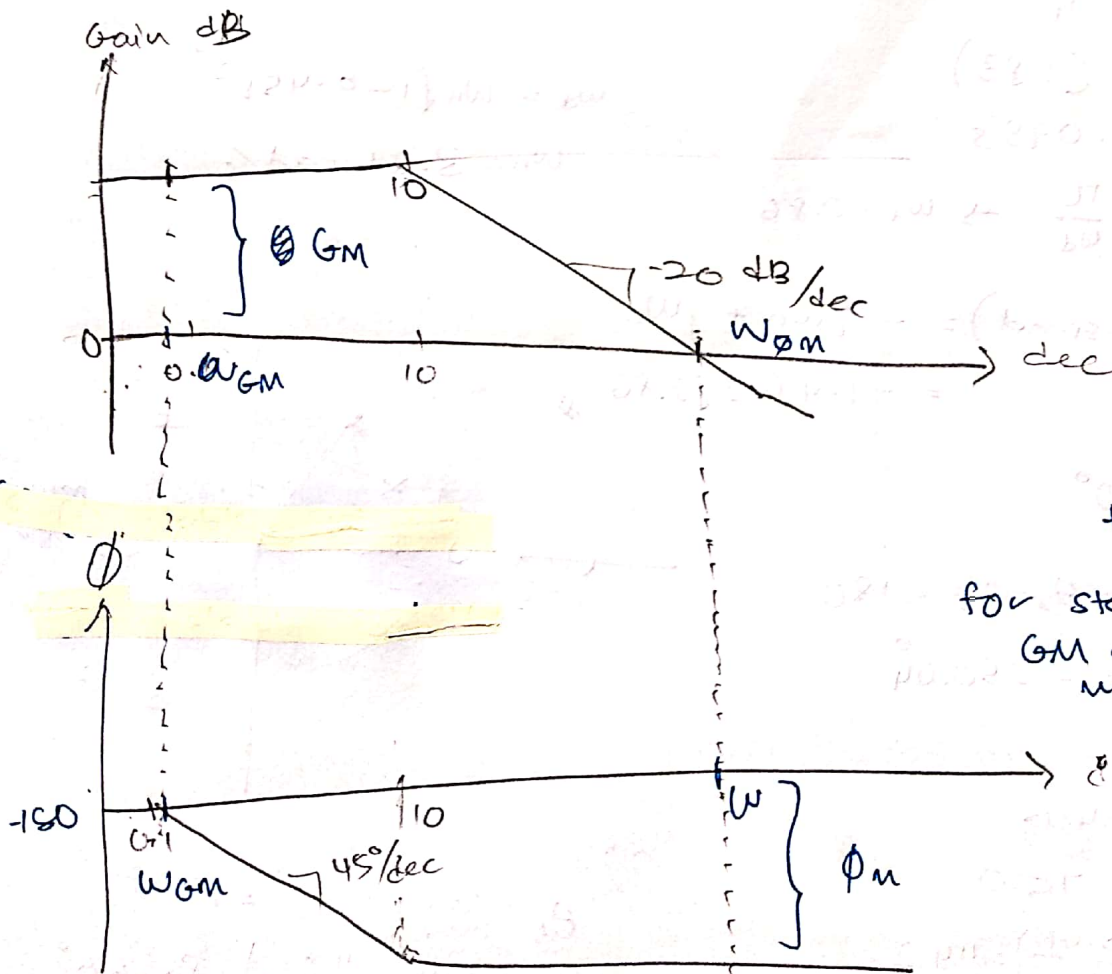
$$G(s) = G(j\omega) = \frac{12}{j\omega(j\omega + 6)}$$

$$= \frac{12}{j20(j20 + 6)}$$

$$= -0.0275 - j8.26 \times 10^{-3}$$

$|G(j\omega)| = 0.028$ #

(b) $G(s) = \frac{100}{(s+10)} = \frac{100}{10(\frac{s}{10} + 1)}$



this is unstable.
for stable;
GM and ϕ_m
must positive.