

Question 1

(a) (i) $G(s) = \frac{(s+3)}{(s+4)(s^2+4s+5)}$

Zero :-
 $s = -3$

Pole :-
 $s = -4, -2 \pm j1$

(ii) Asymptote point, σ_a

$$\sigma_a = \frac{[-4 - (2-j1) - (2+j1)] - (-3)}{3-1}$$

= 0

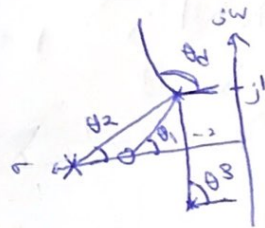
Angle of Asymptote

$$= \frac{(2k+1)180}{2}$$

= $90^\circ, 270^\circ$

(iii) Angle of departure.

$$\theta_d = 45 - 26.57 - 90 + 180 = 108.43$$



b) (i) OS% = 10%

$\zeta = 0.39$

$\omega_n = 4.08 \text{ rad/s}$

Dominant CL pole = $-\zeta\omega_n + j\omega_d = -2.4 + j3.3$

$\zeta\omega_n = 0.39 \times 4.08$
 $= 2.4$

$\omega_d = \omega_n \sqrt{1 - \zeta^2}$
 $= 3.3$

(ii) Gain, k

~~$k = \frac{|s+4||s^2+4s+5|}{|s+3|}$~~ , $s = -2.4 + j3.3$

~~$\frac{8.08 \times 37.97}{7.08}$~~

~~$= 43.33$~~

$k = \frac{|s+4||s+2-j||s+2+j|}{|s+3|}$

$= \frac{8.08 \times 18.66}{7.08}$

$= 21.3$

(iii)

Third pole = -7

Second approximation not valid.

(iv) $T_s = \frac{4}{|\text{Rea}|}$

$= \frac{4}{2.4}$

$= 1.67$

$T_p = \frac{\pi}{3.3}$

$= 0.955$

$e_{ss} = \frac{1}{1+k_p}$

$= \frac{1}{1+3.195}$

$= 0.24$

$k_p = \lim_{s \rightarrow 0} kG(s)$

$= 3.195$

Question 2

$$G(s) = \frac{k(s^2 - 2s + 8)}{(s+3)(s^2 + 4s + 5)}$$

$$s_d = -0.88 \pm j1.72, \text{ OS \%} = 20$$

(a) PD controller.

$$\text{OS \%} = 20\%$$

$$T_s = \frac{4}{|\text{Real}|} = \frac{4}{0.88} = 4.55 \text{ s}$$

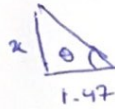
$$\zeta = \frac{-\ln(0.2)}{\sqrt{\pi^2 + \ln^2(0.2)}}$$

$$T_{s \text{ new}} = \frac{3}{5} \times 4.55 = 2.73 \text{ s}$$

$$= 0.46$$

$$\theta = \cos^{-1}(0.46) = 62.6^\circ$$

$$2.73 = \frac{4}{|\text{Real}|}$$



$$\text{Real} = \frac{4}{2.73} = 1.47$$

$$\Rightarrow \tan 62.6 = \frac{2}{1.47} \Rightarrow \alpha = 2.84$$

$$s_d \text{ new} = -1.47 + j2.84$$

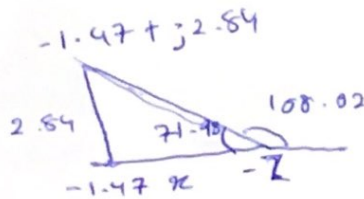
$$G_{PD} = s+2$$

$$G_{PD}G = \frac{k(s+2)(s^2 - 2s + 8)}{(s+3)(s^2 + 4s + 5)}$$

$$\angle s+2 + \angle s-1+j\sqrt{7} + \angle s-1-j\sqrt{7} - \angle s+3 - \angle s+2-j1 - \angle s+2+j1 = 180$$

$$\angle s+2 + \angle 114.24 + \angle 175.5 - \angle 61.69 - \angle 73.93 - \angle 82.14 = 180$$

$$\angle s+2 = 108.02$$



$$\alpha = \frac{2.84}{\tan 71.98}$$

$$\alpha = 0.92$$

$$-Z = -0.55$$

$$G_{PD} = s + 0.55$$

$$k = \frac{|s+3||s+2-j1||s+2+j1|}{|s-\cancel{1-j\sqrt{7}}||s-1+\sqrt{7}j|}$$

$$= \frac{|6.2||5.3||5.31|}{|3.44||3.44|}$$

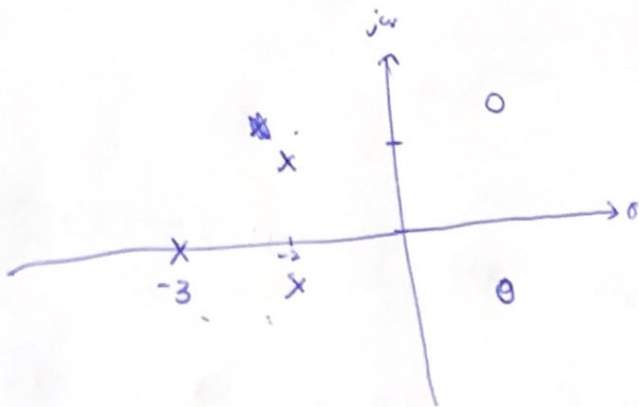
$$= 14.72$$

$$G_{PD}(s) = \frac{14.72(s+0.55)(s^2-2s+8)}{(s+3)(s^2+4s+5)}$$

$$k_p = 0.55 \times 14.72$$
$$= 8.096$$

$$k_d = 14.72$$

b) Compensated



Question 3

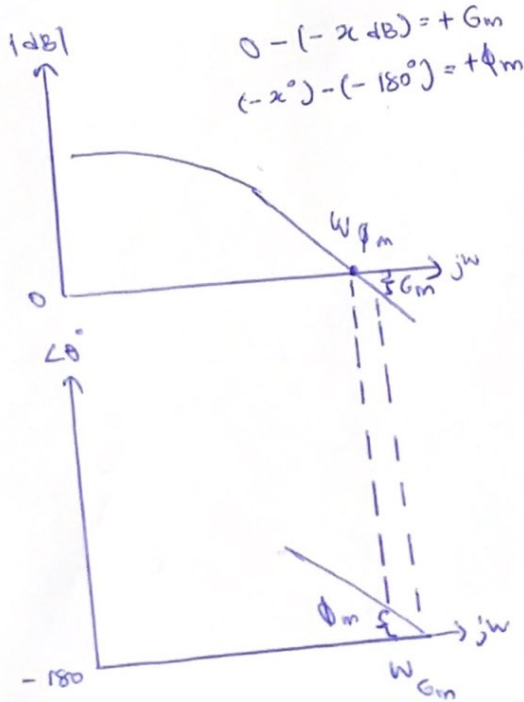
(a) $G(s) = \frac{12}{s(s+b)}$

(i) $|G(j\omega)| = 20 \log 12 - 20 \log |j\omega| - 20 \log |s(j\omega+b)|$

(ii) $|G(j\omega)| = 21.58 - 26.02 - 26.39$
 $= -30.83 \text{ dB}$

(b)

(c) (i) Stable system



(ii) Unstable system

