

Question 1

$$(a) (i) G(s) = \frac{k(s+3)}{(s+4)(s^2+4s+5)}$$

$$(s+2+j)(s+2-j) \\ = s^2+4s+5$$

$$(ii) \phi_a = \frac{(-4-(2+j)-(2-j))-(-3)}{3-1} \\ = -2.5 \#$$

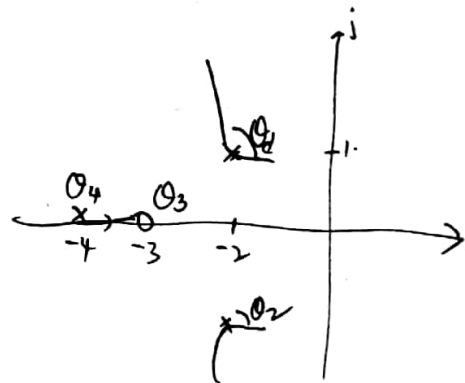
$$\theta_a = \frac{(2k+1)\pi}{2} = \frac{\pi}{2}, \frac{3\pi}{2} \#$$

(iii) Angle of Departure

$$-\theta_d - \theta_2 + \theta_3 - \theta_4 = 180^\circ$$

$$-\theta_d - 90^\circ + \tan^{-1}\left(\frac{1}{1}\right) - \tan^{-1}\left(\frac{1}{2}\right) = 180^\circ$$

$$\theta_d = -251.6^\circ \\ = 108.4^\circ \#$$



$$(b) \zeta = 0.59 \quad \text{OS\%} = 10\% \quad \omega_n = 4.08$$

$$(i) \theta = \cos^{-1} 0.59 \\ = 53.84^\circ$$

$$\text{dominant closed-loop poles} \\ = -2.2 + 1.32j \#$$

$$(ii) K = \frac{L_1 L_2 L_3}{L_4} \\ = 1.27 \#$$

$$L_1 = \sqrt{0.32^2 + 0.32^2} = 0.3771$$

$$L_2 = 2.3286$$

$$L_3 = 2.2321$$

$$L_4 = 1.5435$$

Question 1

b)(iii) Third pole = -4

$$\text{Ratio} = \frac{|-4|}{|-2.2|} = 1.818 < 5$$

∴ This system is not second order approximation.

$$(iv) T_s = \frac{4}{|Re\{s\}|} = \frac{4}{|-2.2|} = 1.818 \text{ s}$$

$$T_p = \frac{\pi}{|Im\{s\}|} = \frac{\pi}{1.32} = 2.38 \text{ s}$$

$$\text{dom. poles} = -2.2 + 1.32j$$

$$K = \frac{(s+4)(s^2+4s+5)}{(s+3)} \quad \begin{array}{l} \text{sub} \\ s = -2.2 + 1.32j \end{array}$$

$$= 1.23$$

$$K_p = \lim_{s \rightarrow 0} \frac{1.23(s+3)}{(s+4)(s^2+4s+5)}$$

$$= 0.1845$$

$$\therefore e_{ss} = \frac{1}{1+K_p} = \frac{1}{1.1845} = 0.8442 \quad \#$$

## Question 2

$$G(s) = \frac{k(s^2 - 2s + 8)}{(s+3)(s^2 + 4s + 5)}$$

dom. pole =  $-0.88 \pm j1.72$  at 20% OS

$$(a) T_{s \text{ old}} = \frac{4}{0.88} = 4.54$$

$$T_{s \text{ new}} = \frac{3}{5} \times 4.54 = 2.73 \text{ s}$$

$$T_{s \text{ new}} = 2.73 = \frac{4}{\xi \omega_n}$$

$$\omega_n = \frac{4}{2.73 \times 0.456} = 3.213$$

$$\text{Real CL-poles} = -\xi \omega_n = -1.465$$

$$\text{Imag CL poles} = j\omega_d = j\omega_n \sqrt{1-\xi^2} = j2.86$$

$$\text{CL poles} = -1.465 + j2.86$$

$$O_s = \left( e^{-\frac{\pi \xi}{\sqrt{1-\xi^2}}} \right) \times 100\% \\ = 20\%$$

$$\xi = 0.456$$

$$T_p = \frac{\pi}{1.72} = 1.826 \text{ s}$$

$$k = \frac{|s+3|(s^2+4s+5)}{|s^2-2s+8|}$$

$$s = -0.88 + j1.72$$

$$= 1.07$$

$$K_v = \lim_{s \rightarrow 0} s G(s) = \lim_{s \rightarrow 0}$$

Question 3

(a)  $G(s) = \frac{12}{s(s+6)}$

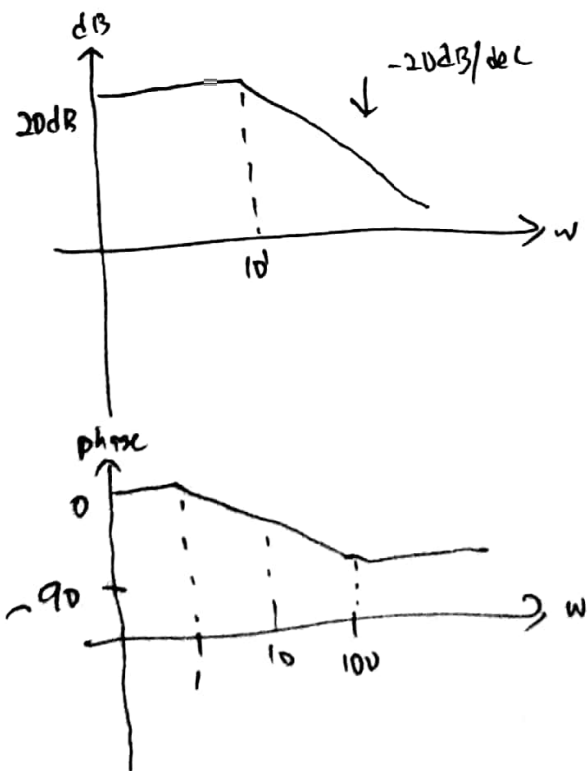
(i)  ~~$20 \log 12$~~   ~~$20 \log$~~

$20 \log |G(j\omega)| = 20 \log 12 - 20 \log |j\omega| - 20 \log |j\omega + 6|$

(ii)  $\omega = 20$

~~$20 \log |G(j\omega)|$~~   $= 32.74 \text{ dB}$

(b)  $G(s) = \frac{100}{s+10} = \frac{100}{10} \frac{1}{\frac{s}{10}+1} = 10 \frac{1}{\frac{s}{10}+1}$

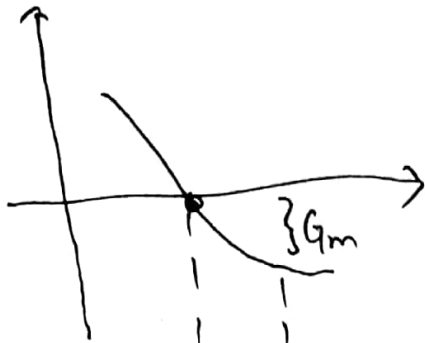


constant = 10 =  $20 \log 10 = 20 \text{ dB}$

Question 3

(c)

(1)



$G_m$  and  $\Phi_m$  is positive

$\therefore$  The system is stable



$G_m$  and  $\Phi_m$  is negative

$\therefore$  The system is unstable

